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ECONOMIC-STATISTICAL DESIGN OF DOUBLE SAMPLING S CONTROL CHART

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Abstract: *In this study, an economic-statistical design of the double sampling (DS) s chart is investigated. The optimal design parameters of the DS s chart are obtained by minimizing the cost subject to statistical constraints. The effectiveness of the DS s chart is evaluated by comparing its minimized cost against the cost of the Shewhart s chart. The comparison demonstrates the superiority of the DS s chart over the Shewhart s chart, where the DS s chart is economically superior to the Shewhart s chart. Sensitivity analyses are performed to investigate the effect of the cost parameters, process parameters and statistical properties on the design parameters, as well as the corresponding in-control average time to signal, out-of-control average time to signal and cost.*

Keywords: *double sampling, economic design, economic-statistical design, s chart*

1. Introduction

Recently, there are many studies on the control chart using double sampling (DS) feature in the literature. Torng et al. (2010) presented the performance of a DS and variable sampling interval $\bar{X}$ chart (DSVSI $\bar{X}$ chart) under non-normality. They compared the DSVSI $\bar{X}$ chart with the Shewhart $\bar{X}$ chart and the variable parameters $\bar{X}$ chart. The comparison results showed that the DSVSI $\bar{X}$ chart has the best performance in detecting small process mean shifts. De Araújo Rodrigues et al. (2011) proposed a DS *np* chart, where the DS *np* chart is the fastest chart for the detection of increases in the fraction non-conforming when compared with the single-sampling *np* chart, the variable sample size *np* chart, the cumulative sum (CUSUM) *np* chart and the exponentially weighted moving average (EWMA) *np* chart. Costa and Machado (2011) presented a pure Markov chain approach to investigate the properties of the variable parameters $\bar{X}$ chart and the DS $\bar{X}$ chart, where it is assumed that the process mean wanders according to the first-order autoregressive model. Faraz et al. (2012) studied the economic statistical design of a DS T^2 chart, where the economic performance of the DS T^2 chart is favourably compared to the multivariate EWMA chart and other variable ratio sampling T^2 control charts in the literature. Lee et al. (2012b) constructed an economic design model of the DSVSI $\bar{X}$ chart for the determination of the design parameters, in which genetic algorithms is used to find the optimal design parameters of the DSVSI $\bar{X}$ chart. Khoo et al. (2013a) studied the effects of parameter estimation on the median run length-based DS $\bar{X}$ chart when the in-control average

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sample size is minimized and their study revealed that more than eighty samples are required for the median run length-based DS $\bar{X}$ chart with estimated parameters to perform favourably with the corresponding known parameters case. Khoo et al. (2013b) evaluated the performance of the DS $\bar{X}$ chart when process parameters are estimated and they showed that performance for the case with estimated parameters is different from that for the corresponding case with known parameters. Lee (2013) proposed a joint DSVSI $\bar{X}$ and s chart, where the comparison study showed that the DSVSI $\bar{X}$ and s chart is able to signal the shifts of both mean and variance better than the joint DS $\bar{X}$ and s chart, the adaptive $\bar{X}$ and R chart, the EWMA chart and the CUSUM chart. Teoh et al. (2013) proposed two optimal designs of the median run length-based DS $\bar{X}$ chart; that are, for minimizing (i) the in-conrol average sample size and (ii) both the in-control and out-of-control average sample size. To overcome the efficiency loss of the median chart, Ahmad et al. (2014) presented a set of auxiliary information-based median type Shewhart charts based on parent normal, t and gamma distributed process environments under the DS scheme. Teoh et al. (2014) studied the DS $\bar{X}$ chart when the process parameters are unknown, where a new optimal design procedure for the DS $\bar{X}$ chart with estimated process parameters is developed to compute the chart's optimal parameters by minimizing the out-of-control median run length. Costa and Machado (2015) investigated the steady-state average run length of the synthetic and side-sensitive synthetic DS $\bar{X}$ charts. They found out that the overall performance of the synthetic DS $\bar{X}$ chart can be improved with the side-sensitive feature but not enough to outperform the non-synthetic DS $\bar{X}$ chart. Teoh et al. (2016) computed the percentiles of the run length distribution for the DS $\bar{X}$ chart with estimated process parameters with information, including the early false alarm, the skewness of the run length distribution and the median run length. Castagliola et al. (2017) investigated the properties of the DS s^2 chart with estimated process variance, in terms of the average run length, the standard deviation of the run length and the average sample size. They also provided guidelines to systematically design the double sampling s^2 chart both with known and estimated process variance. Costa (2017) enhanced the performance of the R chart by using the DS scheme. The trade-off between the operational simplicity and the power of detection of the control chart might lead practitioners to choose the DS R chart, even the DS s^2 chart signalling faster.

The s chart is one of the control charts that have been used widely to monitor shifts in the process standard deviation. Acosta-Mejia and Pignatiello (2009) showed how to design the s charts with k-of-k runs rules, They found out that the s charts that combine the 1-of-1 and k-of-k runs rules with k = 9 or 10 are very effective for monitoring both increases and decreases in the process dispersion. Rakitzis and Antzoulakos (2011) introduced and studied an one-sided adaptive s control chart, supplemented or not with runs rule, for detecting increases or decreases in process variation, where the properties of the proposed control schemes are obtained by using a Markov chain approach. Abbasi and Miller (2012) investigated the performance of various dispersion charts, including s chart for normal and different non-normal processes. They showed that the performance of the s chart is extremely affected for almost all the non-normal processes and the power of the s chart is strongly related to the efficiency of the dispersion estimator used in its construction. Lee et al. (2012a) extended the idea of Carot et al. (2002) by combining the DS s chart and the variable sampling interval s chart. The performance of their proposed chart is compared with the double sampling s chart, the variable sampling interval s chart, the EWMA s chart, and the CUSUM s chart. This comparative study found out that their proposed chart to be efficient in detecting small shifts. Safaei et al. (2012) developed a multi-objective model for the economic-statistical design of the *s* chart by minimizing the mean hourly loss cost as well as minimizing the average time to signal and maintaining reasonable in-control average run length. They showed that the proposed model gives more practical outcomes in comparison with the existing economic design models. Kuo and Lee (2013) extended the idea of adaptive schemes to the *s* chart for improving the signalling increases in the standard deviation. The performances of the adaptive *s* charts are compared with the *s* chart, the *R* chart, the DSVSI *s* chart, the variable parameters *R* chart, the EWMA *s* chart and the CUSUM *s* chart. The results showed that the variable parameters *s* chart is faster in signalling small increases in the standard deviation. Rakitzis and Antzoulakos (2014) introduced and studied an one-sided variable sampling interval *s* chart supplemented with signalling and switching rules based on runs. The proposed one-sided variable sampling interval control charts outperform the corresponding fixed sampling interval schemes in the detection of small increase or decrease shifts in the process variation. Moreover, in most cases, the proposed one-sided variable sampling interval control charts are more effective than other existing improved charting procedures for monitoring the process variation. Zhang (2014) improved the *s* chart with accurate approximation of the control limit by using cumulative distribution function of the sample standard deviation. Their simulation studies showed that the improved *s* chart performs very well. They also compared the type II error risk and average run length of the improved *R* chart and the improved *s* chart, in which they found that the *s* chart is generally more efficient than the *R* chart. Abujiya et al. (2016) proposed a new combined Shewhart-CUSUM *s* chart based on the extreme variations of ranked set sampling technique, for efficient monitoring of changes in the process dispersion. It is observed that there is a great deal of improvements in the performance of the combined Shewhart-CUSUM *s* charts over the individual Shewhart chart and the CUSUM *s* chart. They also found that the proposed schemes outperform certain established schemes in detecting increases and decreases in the process standard deviation. Adeoti and Olaomi (2016) proposed an efficient alternative to the *s* chart for detecting shifts of small magnitude in the process variability by using a moving average based on the sample standard deviation. The performance of the moving average *s* chart is compared to the *s* chart in terms of average run length. The result showed that the performance of moving average *s* chart outweighs those of the *s* chart for small and moderate shifts in the process variability. Lee and Khoo (2017) proposed a synthetic DS *s* chart that integrates the DS *s* chart and the conforming run length chart. The performance of the synthetic DS *s* chart is compared with other existing control charts. The results showed that the synthetic DS *s* chart is effective for detecting increases in the process standard deviation for a wide range of shifts.

Economic-statistical designs for different types of control chart have been recently proposed by many authors. Amiri et al. (2015) used Lorenzen and Vance cost function for the economic and economic-statistical designs of the EWMA chart, in which absolute robustness criterion which minimizes the worst-case scenario and also robust deviation which minimizes the deviation from the optimal solutions are applied to explore the robust approach for the design of the EWMA chart. Chiu (2015) proposed the economic-statistical design of the EWMA chart with time-varying control limits, in which Taguchi’s quadratic loss function is incorporated into the economic-statistical design. Lim et al. (2015) presented the economic and economic-statistical designs of the side sensitive group runs chart based on the average run length and the expected average run length. Nenes et al. (2015) investigated the economic-statistical design of the variable-parameter Shewhart control chart for monitoring the process mean in the presence of multiple assignable causes. They concluded that the occurrence of several assignable causes leads to progressive process deterioration. Safaei et al. (2015) investigated the economic-statistical design of the $\bar{X}$ chart utilising robust optimization approach that considers interval estimates of uncertain parameters. They compared the robust design for an industrial problem with the traditional economic-statistical and heuristic designs. Numerical analyses and simulation study showed that the proposed $\bar{X}$ chart offers a better approach and more reliable solutions for practitioners. Seif et al. (2015) investigated the economic-statistical design of the variable parameters $\bar{X}$ chart when the underling process distribution is non-normal, in which they used the Burr distribution as a model of the process quality variable distribution because of its flexibility in terms of being able to model many distributions including the normal. Yeong et al. (2015) investigated the effects of process parameter estimation on the cost of the synthetic $\bar{X}$ chart. Their study showed that the cost increases when the optimal chart's parameters corresponding to the known process parameters case are used to estimate the cost for the estimated process parameters case. Ershadi et al. (2016) investigated the economic-statistical design of the variable sampling interval profile monitoring. An extended Lorenzen-Vance function is used for modelling the total cost, in which the average time to signal is employed for depicting the statistical measure of the obtained profile monitoring scheme. Faraz et al. (2016) investigated the robust economic-statistical design of the T^2 chart in an attempt to reduce the cost penalties when there are multiple scenarios, where genetic algorithm optimization method is employed to minimize the total expected monitoring cost across all the distinct scenarios. Yeong et al. (2016) proposed the economic and economic-statistical designs of the Hotelling's T^2 chart, where practitioners do not have to specify the Mahalanobis distance shift size (MDSS). They found out that there is a significant increase in cost when adopting optimal design parameters based on the wrong MDSS. Heydari et al. (2017) constructed and compared the economic and economic-statistical designs of the Hotelling's T^2 chart under Burr XII shock models for uniform and non-uniform sampling schemes. Katebi et al. (2017) investigated the economic and economic-statistical designs of the adaptive T^2 chart with two different sampling intervals and three sample sizes, in which they used the Markov chain approach to develop the cost model proposed by Costa and Rahim (2001). Lu and Huang (2017) developed the statistical constraints of the maximum double EWMA chart by applying a loss model that combines Lorenzen and Vance's cost model with linear, quadratic and exponential loss functions. Rafie et al. (2017) developed a method for the economic and economic-statistical designs of the T^2 charts under two-parameter generalized exponential shock model and uniform sampling scheme based on the primary work of Banerjee and Rahim (1988) and Yang and Rahim (2006). Tavakoli et al. (2017) studied the effectiveness of the variable sampling interval scheme on the Bayesian control chart, based on the economic and economic-statistical designs, in which Monte Carlo method and artificial bee colony algorithm are utilized to obtain the optimal design parameters of the Bayesian control chart since the statistic of this approach does not have any specified distribution. Safe et al. (2018) proposed a multi-objective genetic algorithm for the economic-statistical design of the $\bar{X}$ chart with variable sampling interval for identifying the Pareto optimal solutions of the control chart design. Lee and Khoo (2018) studied the economic-statistical design of the synthetic max chart, in which the design of the synthetic max chart is developed by considering the minimization of the cost function, subject to constraints on the average run length.

He and Grigoryan (2002) proposed a DS *s* chart and it is assumed that the distribution of the sample standard deviations follows a normal distribution. After that, He and Grigoryan (2003) developed an improved DS *s* chart without the normality assumption of the sample standard deviation. In this study, an economic-statistical design of the DS *s* chart is presented and its cost function is minimized to obtain the optimal design parameters, subject to statistical constraints by applying the cost model in Lorenzen and Vance (1986). Recently, many authors extended Lorenzen-Vance cost model to develop the economic or economic-statistical design for the control chart. Costa and Fichera (2017) presented the economic-statistical model of an auto-regressive moving average control chart with the design problem consists of minimizing Lorenzen-Vance cost function subject to a constraint on the minimum in-control average run length. Ghanaatiyan et al. (2017) extended Lorenzen-Vance cost function by considering multiple assignable causes and multivariate Taguchi loss approach to obtain the expected cost per unit time for the economic-statistical design of the multivariate EWMA chart with variable sample size and sampling interval. Liu et al. (2017) constructed a minimization of cost based on Lorenzen-Vance economic model to optimize the parameter combination of a modified multivariate EWMA chart. Considering the non-normal input quality characteristics, Patil and Shirke (2017) developed a loss cost function for the moving average control chart under the continuous, ceased and semi-ceased process models by using Lorenzen and Vance unified approach. Salmasnia et al. (in press) applied the cost model offered by Lorenzen and Vance (1986) to design the double warning lines T^2 chart, in which their research aim is to optimize a multi-objective economic-statistical design model by concurrently monitoring the cost and statistical feature of the double warning lines T^2 chart. Seif (in press) employed Lorenzen-Vance cost model to build the economic-statistical design of the T^2 chart with variable sample size, in which the cost model is derived by the Markov chain approach, and genetic algorithms is applied to find the optimal design parameters.

The remainder of the paper is organized as follows. The DS *s* chart is reviewed in Section 2. The economic-statistical design of the DS *s* chart and its cost model are discussed in Section 3 with an example is provided to illustrate the solution procedure. The potential cost saving for monitoring the variability of a process with the DS *s* chart instead of using the Shewhart *s* chart is also investigated in Section 3. Section 3 also compares the economic-statistical design with the economic design. Sensitivity analysis for the effect of the cost and process parameters on the economic-statistical design of the DS *s* chart is carried out in Section 4. Furthermore, sensitivity analysis for the effect of the statistical properties on the economic-statistical design of the DS *s* chart is also performed in Section 4. Finally, concluding remarks are provided in Section 5.

2. A review of the DS s chart

The DS *s* chart has been shown to be faster than the Shewhart *s* chart in detecting process standard deviation shifts (He & Grigoryan, 2003). According to Kuo and Lee (2013), an increase in the process standard deviation indicates inferior production quality, whereas a decrease in the process standard deviation indicates an increase in the quality level. In addition, practitioners are usually more interested in the control of increases in the process variation than in the detection of decreases (Rakitzis & Antzoulakos, 2011). Therefore, the DS *s* chart in this study is designed to signal the increases in the process standard deviation.

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a normally distributed process with mean of μ and standard deviation σ. It is assumed that the initial state of the process has an in-control standard deviation of $\sigma = \sigma_0$. When an increase in the process standard deviation occurs, the value of σ changes from the in-control value of σ_0 to the out-of-control value of $\gamma\sigma_0$, where $\gamma > 1$ is the coefficient of the standard deviation shift. Note that when $\gamma = 1$, there is no shift in the process standard deviation.

Let WL, CL_1 and CL_2 be the warning limit, the control limit at the first stage of the DS scheme and the control limit at the second stage of the DS scheme, respectively. Then, WL $= (c_1 + w\sqrt{1 - c_1^2})\sigma_0$, $\mathrm{CL}_1 = (c_1 + k_1\sqrt{1 - c_1^2})\sigma_0$, $\mathrm{CL}_2 = (c_2 + k_2\sqrt{1 - c_2^2})\sigma_0$. Here, $c_1 = \sqrt{2/(n_1 - 1)}\Gamma(n_1/2)/\Gamma[(n_1 - 1)/2]$ is the coefficient for a sample of size n_1 and $c_2 = \sqrt{2/(n_1 + n_2 - 2)}\Gamma[(n_1 + n_2 - 1)/2]/\Gamma[(n_1 + n_2 - 2)/2]$ is the coefficient for a sample of size $(n_1 + n_2)$, w is the coefficient of WL, k_1 is the coefficient of CL_1, k_2 is the coefficient of CL_2 and $\Gamma(\cdot)$ is the gamma function (He & Grigoryan, 2003). The charting procedure for the DS s chart (He & Grigoryan, 2003) is as follows:

Step 1. Take the first sample of size n_1 and the standard deviation of this sample is calculated as $s_1 = \sqrt{\sum_{i=1}^{n_1}(X_i - \bar{X}_1)^2 / (n_1 - 1)}$, where $\bar{X}_1 = \left(\sum_{i=1}^{n_1} X_i\right)/n_1$, then the process flow goes to the next step.

Step 2. If s_1 < WL, the process is considered to be under control and the control flow returns to Step 1.
If WL < s_1 < CL_1, take the second sample of size n_2 and the standard deviation of this sample is calculated as $s_2 = \sqrt{\sum_{i=1}^{n_2}(X_i - \bar{X}_2)^2 / (n_2 - 1)}$, where $\bar{X}_2 = \left(\sum_{i=1}^{n_2} X_i\right)/n_2$, then the total sample standard deviation is calculated as $s_{12} = \sqrt{\left((n_1 - 1)s_1^2 + (n_2 - 1)s_2^2\right)/(n_1 + n_2 - 2)}$ and the control flow goes to the next step.
If s_1 > CL_1, the process is considered to be out-of-control and the process flow goes to Step 4.

Step 3. If s_{12} < CL_2, the process is considered to be under control and the process flow return to Step 1. Otherwise, the process is considered to be out-of-control and the process flow goes to the next step.

Step 4. Corrective action is taken to remove the assignable cause, then the process flow return to Step 1.

The probability for a sample to fall below the control limits (i.e. $s_1 < \mathrm{CL}_1$ or $s_{12} < \mathrm{CL}_2$) is calculated as

$$\rho = \chi^2_{n_1 - 1}\left(\frac{(n_1 - 1)\mathrm{WL}^2}{\gamma^2\sigma_0^2}\right) + \int_{\xi_1}^{\xi_2} \chi^2_{n_2 - 1}\left(\frac{(n_1 + n_2 - 2)\mathrm{CL}_2^2}{\gamma^2\sigma_0^2} - \omega\right) f_{n_1 - 1}(\omega)\,d\omega, \tag{1}$$

where $\xi_1 = (n_1 - 1)\mathrm{WL}^2/(\gamma\sigma_0)^2$, $\xi_2 = (n_1 - 1)\mathrm{CL}_2^2/(\gamma\sigma_0)^2$, $\chi^2_v(\cdot)$ and $f_v(\cdot)$ are the cumulative distribution function and the probability density function of a chi-square distribution with v degrees of freedom, respectively (Lee et al., 2012a).

The average sample size is $\bar{n} = n_1 + n_2\{\chi^2_{n_1 - 1}[(n_1 - 1)\mathrm{CL}_1^2/(\gamma\sigma)^2] - \chi^2_{n_1 - 1}[(n_1 - 1)\mathrm{WL}^2/(\gamma\sigma)^2]\}$ (Lee et al., 2012). Here, $\bar{n} = \bar{n}_0$ when $\gamma = 1$ and $\bar{n} = \bar{n}_\gamma$ when $\gamma > 1$. The average run length of the DS s chart is given as ARL = $1/(1 - \rho)$. Consequently, the average time to signal of the DS *s* chart is computed as ATS = ARL × *h*, where *h* is the sampling interval.

3. Economic-statistical design of the DS s chart

3.1. Cost model

Lorenzen and Vance (1986) proposed a general cost model for designing the economic model of the control chart, in which their proposed cost model can be applied to all the control charts regardless of the statistic used. Consequently, this study develops an economic-statistical design of the DS *s* chart, in which the cost model is based on the cost function in Lorenzen and Vance (1986). The assumptions of the cost model are as follows:

1) The process begins in an in-control state and follows a normal distribution with mean μ_0 and standard deviation σ_0.
2) An assignable cause of variation exists at a random time and causes a shift in the process standard deviation.
3) The assignable cause is assumed to occur according to a Poisson process with λ occurrences per hour.
4) The quality cycle follows a renewal reward process.

Based on the cost model in Lorenzen and Vance (1986), the expected cost per hour is given as:

$$C = \frac{C_0/\lambda + C_1(-\tau + \bar{n}_\gamma e + \mathrm{ATS}_1 + r_1T_1 + r_2T_2) + sY/\mathrm{ARL}_0 + W}{1/\lambda + (1-r_1)sT_0/\mathrm{ARL}_0 - \tau + \bar{n}_\gamma e + \mathrm{ARL}_1 h + T_1 + T_2} + \frac{(a + b\bar{n}_0)s + (\frac{a + b\bar{n}_\gamma}{h})\left(\bar{n}_\gamma e + \mathrm{ATS}_1 + r_1T_1 + r_2T_2\right)}{1/\lambda + (1-r_1)sT_0/\mathrm{ARL}_0 - \tau + \bar{n}_\gamma e + \mathrm{ATS}_1 + T_1 + T_2} \quad (2)$$

where ARL_0 is the in-control ARL; ARL_1 is the out-of-control ARL; λ is the expected time of occurrence of an assignable cause; C_0 is the cost of producing non-conformities while the process is in-control; C_1 is the cost of producing non-conformities while the process is out-of-control; W is the cost to locate and repair an assignable cause; a is the fixed cost per sample; and b is the variable cost per unit sampled; Y is the cost per false alarm; T_0 is the expected time spent searching for a false alarm; T_1 is the expected time to discover an assignable cause; T_2 is the expected time to repair the process; e is the time to sample and chart one item; r_1 = 1 if the production continues during searches and 0 otherwise; r_2 = 1 if the production continues during repairs and 0 otherwise; $\tau = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(1-e^{-\lambda h})}$ is the expected time between the occurrence of an assignable cause and the previous sample; $s = \frac{e^{-\lambda h}}{1-e^{-\lambda h}}$ is the expected number of samples taken when the process is in-control.

The cost model of the DS *s* chart is designed with respect to statistical criteria such that $\mathrm{ATS}_0 \geq \mathrm{ATS}_L$ and $\mathrm{ATS}_1 \leq \mathrm{ATS}_U$, where ATS_0 is the in-control ATS and ATS_1 is the out-of-control ATS. Here, the ATS_L is the lower bound of the ATS_0 and the ATS_U is the upper bound of the ATS_1, where the ATS_L and ATS_U are the pre-determined values of the statistical constraints. Here, the ATS_L is the bound of the in-control statistical performance which provides protection against the false alarm, while the ATS_U is the bound of the out-of-control statistical performance which is to detect the process shift as quickly as possible.

In this study, an add-in software to Microsoft Excel called Evolver is used to determine the optimal design parameters (n_1, n_2, w, k_1, k_2 and h) as well as the corresponding ATS_0, ATS_1 and C values by minimizing the cost function in Equation (2) with the statistical constraints, for the given values of the cost and process parameters (λ, C_0, C_1, W, a, b, Y, T_0, T_1, T_2, e, r_1 and r_2).

3.2. An illustrative example

A numerical example taken from Lorenzen and Vance (1986) is given here to illustrate the economic-statistical design of the DS s chart.

The cost and process parameters are λ = 0.02, C_0 = 114.24, C_1 = 949.2, W = 1086, a = 0, b = 4.22, Y = 977.4, $T_0 = T_1 = e$ = 5/60, T_2 = 15/60 with r_1 = 1 and r_2 = 0. The maximum values of the sample size and sampling interval are set as n_2 = 20 and h = 3, respectively. The DS s chart is designed with respect to $ATS_0 \geq 370.4$ and $ATS_1 \leq 3$ subject to $n_1 < n_2 \leq 20$, $0 < h \leq 3$ and $0 < w < k_1$, where n_1 and n_2 are positive integers.

Table 1 presents the performance comparison between the economic design (ED) and economic-statistical design (ESD) for different shifts γ.

Table 1. Comparisons of ATS_0, ATS_1 and cost between economic and economic-statistical designs of the DS s chart at different shifts for the illustrative example.

γ	ATS_0		ATS_1		C		Percentage of increase in ATS_0	Percentage of increase in ATS_1	Percentage of increase in C
	ED	ESD	ED	ESD	ED	ESD			
1.2	34.3859	370.4	4.9634	3.0000	270.4984	559.5455	977.19	-39.56	106.8572
1.4	105.9967	370.4	3.0273	3.0000	224.4010	234.0118	249.44	-0.90	4.28
1.6	189.9249	370.4	2.3397	2.4161	204.0147	205.1698	95.02	3.27	0.57
1.8	277.3234	370.4	2.0045	2.0994	192.7877	193.1780	33.56	4.73	0.20
2.0	362.6755	370.4	1.8135	1.8143	185.6983	185.6989	2.13	0.04	0.00
2.2	468.9037	468.9037	1.6877	1.8143	180.8131	180.8131	0.00	0.00	0.00
2.4	563.3010	563.3010	1.6051	1.6051	177.2734	177.2734	0.00	0.00	0.00
2.6	732.7006	732.7006	1.5391	1.5391	174.5740	174.5740	0.00	0.00	0.00

The percentage of increases in the ATS_0, ATS_1 and C are calculated as:

$$\frac{\text{ATS}_0 \text{ of ESD} - \text{ATS}_0 \text{ of ED}}{\text{ATS}_0 \text{ of ED}} \times 100\%, \quad (3)$$

$$\frac{\text{ATS}_1 \text{ of ESD} - \text{ATS}_1 \text{ of ED}}{\text{ATS}_1 \text{ of ED}} \times 100\% \quad (4)$$

and

$$\frac{C \text{ of ESD} - C \text{ of ED}}{C \text{ of ED}} \times 100\%, \quad (5)$$

respectively.

The results in Table 1 indicate that at a small γ, the ESD significantly gives a larger ATS_0, a smaller ATS_1 and a higher C compared to the ED. This means that at the small shifts, the statistical performances of the ESD are significantly improved but more costly compared to the ED. For example, for γ = 1.2, the ATS_0 of the ESD is increased by 977.19%, while the ATS_1 of the ESD is decreased by 39.56% with the percentage of an increase in C as 106.86%. For the moderate shifts (i.e $1.6 \leq \gamma \leq 1.8$), the ESD gives larger values in ATS_0 and ATS_1 with a slightly higher value in C compared to the ED. It can be noticed that for shifts of magnitude γ = 2.2 or larger, there is no difference between the ESD and the ED.

3.3. Performance comparisons

The input parameters for the numerical comparisons in this sub-section are the cost and process parameters, i.e. λ, C_0, C_1, W, a, b, Y, T_0, T_1, T_2 and e. Each of these parameters has three levels as shown in Table 2. An experimental design is used to assign these eleven input parameters to L36 orthogonal array. In this orthogonal array experimental design, there are 36 cases, as shown in Table 3.

Table 2. Level plan for the cost and process parameters.

Parameters		λ	C_0	C_1	W	a	b	Y	T_0	T_1	T_2	e
Level	1	0.01	50	500	500	1	2	500	0.05	0.05	0.5	0.05
	2	0.02	114.24	949.2	1086	2	4.22	977.4	0.08333	0.08333	0.75	0.08333
	3	0.05	200	1500	1500	5	10	1500	0.15	0.15	1	0.15

Table 3. The experimental design of the L36 orthogonal array.

Case	λ	C_0	C_1	W	a	b	Y	T_0	T_1	T_2	e
1	0.01	50	500	500	1	2	500	0.05	0.05	0.5	0.05
2	0.02	114.24	949.2	1086	2	4.22	977.4	0.08333	0.08333	0.75	0.08333
3	0.05	200	1500	1500	5	10	1500	0.15	0.15	1	0.15
4	0.01	50	500	500	2	4.22	977.4	0.08333	0.15	1	0.15
5	0.02	114.24	949.2	1086	5	10	1500	0.15	0.05	0.5	0.05
6	0.05	200	1500	1500	1	2	500	0.05	0.08333	0.75	0.08333
7	0.01	50	949.2	1500	1	4.22	1500	0.15	0.05	0.75	0.08333
8	0.02	114.24	1500	500	2	10	500	0.05	0.08333	1	0.15
9	0.05	200	500	1086	5	2	977.4	0.08333	0.15	0.5	0.05
10	0.01	50	1500	1086	1	10	977.4	0.15	0.08333	0.5	0.08333
11	0.02	114.24	500	1500	2	2	1500	0.05	0.15	0.75	0.05
12	0.05	200	949.2	500	5	4.22	500	0.08333	0.05	1	0.15
13	0.01	114.24	1500	500	5	4.22	500	0.15	0.15	0.75	0.05
14	0.02	200	500	1086	1	10	977.4	0.05	0.05	1	0.08333
15	0.05	50	949.2	1500	2	2	1500	0.08333	0.08333	0.5	0.15
16	0.01	114.24	1500	1086	1	2	1500	0.08333	0.15	1	0.08333
17	0.02	200	500	1500	2	4.22	500	0.15	0.05	0.5	0.15
18	0.05	50	949.2	500	5	10	977.4	0.05	0.08333	0.75	0.05
19	0.01	114.24	500	1500	5	10	500	0.08333	0.08333	0.5	0.08333
20	0.02	200	949.2	500	1	2	977.4	0.15	0.15	0.75	0.15
21	0.05	50	1500	1086	2	4.22	1500	0.05	0.05	1	0.05
22	0.01	114.24	949.2	1500	5	2	977.4	0.05	0.05	1	0.15
23	0.02	200	1500	500	1	4.22	1500	0.08333	0.08333	0.5	0.05
24	0.05	50	500	1086	2	10	500	0.15	0.15	0.75	0.08333
25	0.01	200	949.2	500	2	10	1500	0.05	0.15	0.5	0.08333
26	0.02	50	1500	1086	5	2	500	0.08333	0.05	0.75	0.15
27	0.05	114.24	500	1500	1	4.22	977.4	0.15	0.08333	1	0.05
28	0.01	200	949.2	1086	2	2	500	0.15	0.08333	1	0.05
29	0.02	50	1500	1500	5	4.22	977.4	0.05	0.15	0.5	0.08333
30	0.05	114.24	500	500	1	10	1500	0.08333	0.05	0.75	0.15
31	0.01	200	1500	1500	2	10	977.4	0.08333	0.05	0.75	0.05
32	0.02	50	500	500	5	2	1500	0.15	0.08333	1	0.08333
33	0.05	114.24	949.2	1086	1	4.22	500	0.05	0.15	0.5	0.15
34	0.01	200	500	1086	5	4.22	1500	0.05	0.08333	0.75	0.15
35	0.02	50	949.2	1500	1	10	500	0.08333	0.15	1	0.05
36	0.05	114.24	1500	500	2	2	977.4	0.15	0.05	0.5	0.08333

3.3.1. Comparison between the DS s chart and the Shewhart s chart

An investigation is undertaken to explore the potential savings for monitoring the variability of a process with the DS *s* chart instead of using the Shewhart *s* chart. For each of the cases given in Table 3, the outputs (i.e. optimal design parameters as well as the corresponding ATS_0, ATS_1 and C) of the economic-statistical design for the DS *s* chart are listed in Table 4 (see Appendix). The outputs of the economic-statistical design for the Shewhart *s* chart are also provided in Table 4 for comparison purposes. Here, $ATS_L = 370.4$, $ATS_U = 3$ and $\gamma = 1.5$. The last column of Table 4 is the percentage of a decrease in C and it is calculated as:

$$\frac{C_S - C_{DS}}{C_S} \times 100\ \% \tag{6}$$

where C_S and C_{DS} denote the minimum values of C for the Shewhart *s* chart and the DS *s* chart, respectively. Here, the percentage of a decrease in C is the percentage of a reduction in the cost achieved by the DS *s* chart in comparison to the cost associated with the Shewhart *s* chart. The average cost of a reduction by the DS *s* chart against the Shewhart *s* chart is 12.59% with the percentage of saving ranges from 2.97% to 25.09%, where the DS *s* chart outperforms the Shewhart *s* chart in all the given cases. Therefore, a DS *s* chart is preferred to a Shewhart *s* chart for process monitoring.

3.3.2. Comparison between the DS s chart and the Shewhart s chart

During the optimization of the cost function for the economic-statistical design of the DS *s* chart, statistical constraints are imposed such that $ATS_0 \geq 370.4$ and $ATS_1 \leq 3$, to assure that the statistical properties can be attained. In Table 5 (see Appendix), the economic-statistical design of the DS *s* chart is compared to the economic design of the DS *s* chart, where the percentage of increases in ATS_0, ATS_1 and C are calculated using Equations (3), (4) and (5), respectively. When considering the economic-statistical design instead of the economic design, on average, the percentage of an increase in the ATS_0 is 373.15% and the percentage of a decrease in the ATS_1 is 13.87%, with the percentage of an increase in C as 4.30%. These results show that although the economic design has a slightly lower cost saving compared with the economic-statistical design but the economic-statistical design gives a significantly lower false alarm rate and it is also faster in detecting standard deviation shifts in comparison with the economic design, due to the imposed statistical constraints on the ATS_0 and ATS_1.

4. Sensitivity analysis

Specifying the values of the cost and process parameters might be a difficult task when implementing the economic-statistical design of the DS *s* chart. To resolve this problem, sensitivity analysis can be carried out by studying the effect of the cost and process parameters on the economic-statistical design of the DS *s* chart. In this section, two sensitivity analyses (i.e. the economic sensitivity analysis and the statistical sensitivity analysis) are conducted by varying the input parameters, i.e. the cost and process parameters.

4.1. Economic sensitivity analysis for the effect of cost and process parameters on the economic-statistical design of the DS s chart

The economic sensitivity analysis for the economic-statistical design of the DS *s* chart is carried out by using the experimental design in Table 3, where Table 2 illustrates the level plan for each of the input parameters in the sensitivity analysis, in which the values cover a reasonable complete range for each of the input parameters so that the effects of these parameters on the economic-statistical design of the DS *s* chart can be fully examined.

The output parameters n_1, n_2, w, k_1, k_2, h, ATS_0, ATS_1 and C are provided in Table 4 under the column "DS *s* chart". Tables 6-13 show the outputs of the multiple regression analysis with stepwise selection method based on the hypothesis testing for the economic sensitivity analysis.

Table 6 shows the outputs of regression analysis for n_2 of the economic-statistical design for the DS *s* chart. From the ANOVA results in Table 6, there are at least one of the input parameters that significantly affect n_2 at the 5% level of significance since the *p*-value is 0.0372 (less than 0.05). From the table of regression coefficients in Table 6, n_2 is significantly affected by the parameter W only, where a larger value of W results in a larger value of n_2 since the coefficient of the parameter W is a positive value.

Table 6. Output of the stepwise regression analysis for the economic sensitivity of n_2 to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	F	p-value
Regression	3.7363	1	3.7363	4.7025	0.0372
Residual	27.0138	34	0.7945		
Total	30.75	35			

Table of regression coefficients

Parameters	Coefficient	Standard error	t	p-value
Constant	18.6089	0.4010	46.4031	2.47E-32
W	0.0008	0.0004	2.1685	0.0372

Table 7 shows the outputs of the regression analysis for w of the economic-statistical design for the DS *s* chart. The value of w tends to be larger when the parameter e, C_1 or λ is large. However, the value of w tends to be smaller when the parameter a or b is large.

Table 7. Output of the stepwise regression analysis for the economic sensitivity of w to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	F	p-value
Regression	1.4787	5	0.2957	36.7189	6.51E-12
Residual	0.2416	30	0.0081		
Total	1.7203	35			

Table of regression coefficients

Parameters	Coefficient	Standard error	t	p-value
Constant	0.97643	0.0657	14.8549	2.27E-15
e	2.9521	0.3656	8.0755	5.16E-09
C_1	0.0003	3.66E-05	7.3624	3.35E-08
λ	5.1664	0.8850	5.8378	2.2E-06
a	-0.0311	0.0089	-3.5121	0.0014
b	-0.0163	0.0044	-3.6694	0.0009

Table 8 shows the outputs of regression analysis for k_1 of the economic-statistical design for the DS *s* chart. From the results in this table, the value of k_1 will be larger when the parameter λ or C_1 increases.

Table 8. Output of the stepwise regression analysis for the economic sensitivity of k_1 to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	*F*	*p*-value
Regression	3.084652	2	1.542326	5.092026	0.011821
Residual	9.995385	33	0.30289		
Total	13.08004	35			

Table of regression coefficients

Parameters	Coefficient	Standard error	*t*	*p*-value
Constant	3.4498	0.2788	12.3728	6.09E-14
λ	12.3530	5.3967	2.2890	0.0286
C_1	0.0005	0.0002	2.2236	0.0331

Table 9 shows the outputs of regression analysis for k_2 of the economic-statistical design for the DS *s* chart. Increases in the parameter λ or C_1 result in a larger k_2; but increases in the parameter *b* result in a smaller k_2.

Table 9. Output of the stepwise regression analysis for the economic sensitivity of k_2 to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	*F*	*p*-value
Regression	0.4834	3	0.1611	31.4415	1.16E-09
Residual	0.1640	32	0.0051		
Total	0.6474	35			

Table of regression coefficients

Parameters	Coefficient	Standard error	*t*	*p*-value
Constant	2.5623	0.0410	62.489	5.1E-35
λ	4.0745	0.7020	5.804	1.91E-06
b	-0.0200	0.0035	-5.6456	3.04E-06
C_1	0.0002	2.92E-05	5.3630	6.91E-06

Table 10 shows the outputs of regression analysis for *h* of the economic-statistical design for the DS *s* chart. A higher value in *b* leads to a longer *h*; whereas a higher value in C_1, *e* or λ leads to a shorter *h*.

Table 10. Output of the stepwise regression analysis for the economic sensitivity of *h* to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	*F*	*p*-value
Regression	3.9389	4	0.9847	8.3502	0.0001
Residual	3.6558	31	0.1179		
Total	7.5947	35			

Table of regression coefficients

Parameters	Coefficient	Standard error	t	p-value
Constant	1.7675	0.2399	7.3688	2.69E-08
C_1	-0.0005	0.0001	-3.6386	0.0010
e	-3.8773	1.3910	-2.7875	0.0090
λ	-8.0609	3.3862	-2.3805	0.0236
b	0.0390	0.0170	2.2912	0.0289

Table 11 shows the outputs of regression analysis for ATS_0 of the economic-statistical design for the DS s chart. An increase in Y gives a longer ATS_0. On the other hand, an increase in λ, b or C_1 gives a shorter ATS_0.

Table 11. Output of the stepwise regression analysis for the economic sensitivity of ATS_0 to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	F	p-value
Regression	539411.7	4	134852.9	42.9410	3.22E-12
Residual	97353.26	31	3140.428		
Total	636764.9	35			

Table of regression coefficients

Parameters	Coefficient	Standard error	t	p-value
Constant	273.0178	39.3131	6.9447	8.63E-08
Y	0.1898	0.0229	8.3010	2.24E-09
λ	-3413.83	549.5127	-6.2125	6.71E-07
b	-16.4879	2.7698	-5.9527	1.4E-06
C_1	-0.1226	0.0228	-5.3691	7.45E-06

Table 12 shows the outputs of regression analysis for ATS_1 of the economic-statistical design for the DS s chart. The value of ATS_1 will be smaller as the parameter C_1 or λ increases. However, the value of ATS_1 will be larger as the parameter b or C_0 increases.

Table 12. Output of the stepwise regression analysis for the economic sensitivity of ATS_1 to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	F	p-value
Regression	98.6284	4	24.6571	44.7051	1.91E-12
Residual	17.0980	31	0.5516		
Total	115.7265	35			

Table of regression coefficients

Parameters	Coefficient	Standard error	t	p-value
Constant	5.0484	0.4907	10.2885	1.62E-11
C_1	-0.0024	0.0003	-7.9204	6.1E-09
b	0.2932	0.0367	7.9866	5.12E-09
λ	-50.0769	7.2824	-6.8764	1.04E-07
C_0	0.0045	0.0020	2.2399	0.0324

Table 13 shows the outputs of regression analysis for C of the economic-statistical design for the DS s chart. The changes in parameter λ, C_0, C_1, b, W or e significantly affect C, in which the sign of the coefficient of all these parameters are positive, indicating that an increase in one of these parameters gives a higher value in C.

Table 13. Output of the stepwise regression analysis for the economic sensitivity of C to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	F	p-value
Regression	253394.7	6	42232.45	133.13	9.39E-20
Residual	9199.585	29	317.2271		
Total	262594.3	35			

Table of regression coefficients

Parameters	Coefficient	Standard error	t	p-value
Constant	-88.1197	15.5895	-5.6525	4.15E-06
λ	2752.228	175.6429	15.6695	1.08E-15
C_0	0.9287	0.0486	19.1061	5.65E-18
C_1	0.0677	0.0073	9.3116	3.24E-10
b	6.6574	0.883	7.5398	2.6E-08
W	0.0242	0.0073	3.3407	0.0023
e	228.1629	72.7343	3.1369	0.0039

Note that the outputs of the regression analysis for n_1 in Table 14 show that n_1 is not significantly affected by any of the cost and process parameters at the 5% level of significance since the results show the p-value of more than 0.05.

Table 14. Output of the stepwise regression analysis for the economic sensitivity of n_1 to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	F	p-value
Regression	66.1366	11	6.0124	0.8348	0.6092
Residual	172.8634	24	7.2026		
Total	239.0	35			

The factor's responses for the economic sensitivity analysis on C are shown in Table 15. For instance, $\lambda = 0.01$ at Level 1 (see Table 2) is assigned to the 1st, 4th, 7th, 10th, 13th, 16th, 19th, 22nd, 25th, 28th, 31st, 34th cases in Table 3, then the mean value of C for $\lambda = 0.01$ at Level 1 (denoted as $\lambda 1$) is calculated as $\lambda 1 = (C_1 + C_4 + C_7 + C_{10} + C_{13} + C_{16} + C_{19} + C_{22} + C_{25} + C_{28} + C_{31} + C_{34})/12 = (82.31 + 102.35 + 124.29 + 172.94 + 197.35 + 184.45 + 207.32 + 180.46 + 295.59 + 246.37 + 317.06 + 254.62)/12 = 197.09$, where C_i is the value of C_{DS} for the ith case in Table 4. Similarly, the mean values of C for parameter λ at Level 2 ($\lambda = 0.02$) and Level 3 ($\lambda = 0.05$) are $\lambda 2 = 232.47$ and $\lambda 3 = 311.6$, respectively. Then, the effect of λ is $311.60 - 197.09 = 114.51$, determined by the range of values for $\lambda 1$, $\lambda 2$ and $\lambda 3$. The mean values of C for the parameters λ, C_0, C_1, b, W and e are depicted in Figure 1, where C is significantly affected by these parameters. From Figure 1, it is shown that the combination of the optimal input parameter levels that gives the minimum C is $\lambda 1$, $C_0 1$, $C_1 1$, $b1$, $W1$ and $e2$.

Table 15. Factor responses for the economic sensitivity analysis on C.

Parameters		λ	C_0	C_1	W	a	b	Y	T_0	T_1	T_2	e
Level	1	197.09	181.83	213.81	236.76	240.59	224.16	248.33	253.41	243.54	246.40	238.89
	2	232.47	236.26	246.99	244.16	247.56	240.72	239.37	241.62	247.47	242.70	228.27
	3	311.60	323.07	280.36	260.24	253.02	276.29	253.47	246.13	250.15	252.07	274.00
Effect		114.51	141.24	66.55	23.47	12.43	52.12	14.10	11.79	6.61	9.37	45.73
Ranking		2	1	3	6	8	4	7	9	11	10	5

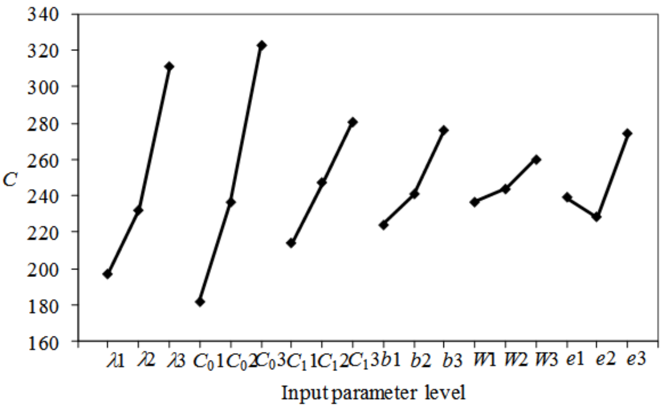

Figure 1. Effect of input parameters on C for the economic-sensitivity analysis.

A normal probability plot of the standardized residual for the effect on C based on the economic sensitivity analysis is plotted as shown in Figure 2, for the model diagnostic checking. It can be noticed that the points plotted on this graph reasonably lie close to a straight line, revealing that the underlying assumption of the analysis is satisfied.

4.2. Statistical sensitivity analysis for the effect of statistical properties on the economic-statistical design of the DS s chart

The statistical sensitivity analysis for the economic-statistical design of the DS s chart is conducted to investigate the effect of statistical properties (i.e. γ, ATS_L and ATS_U) on the design parameters as well as the corresponding ATS_0, ATS_1 and C. The input parameters γ, ATS_L and ATS_U and their three-level parameters are given in Table 16. The input parameters for the L9 orthogonal array and the output parameters (i.e. n_1, n_2, w, k_1, k_2, h, ATS_0, ATS_1 and C) are listed in Table 17.

The statistical sensitivity analysis is examined by the multiple regression analysis and the results are provided in Tables 18-26. The results in Tables 18-21 show that the changes in γ significantly affect the parameters n_1, n_2, k_1 and h. A larger γ results in smaller values of n_1, n_2, and k_1 with longer h.

Table 16. Levels of each input parameter for the statistical sensitivity analysis.

Parameters		γ	ARL_L	ARL_U
Level	1	1.2	250	2
	2	1.5	370.40	3
	3	2	480	4

Table 17. The experimental design of the L9 orthogonal array for the statistical sensitivity analysis.

Case	γ	ARL_L	ARL_U	n_1	n_2	w	k_1	k_2	h	ATS_0	ATS_1	C
1	1.2	250	2	19	20	5.0415	1.3814	3.339	0.1606	250	2	722.92
2	1.2	370.4	3	19	20	4.9626	1.4026	3.3311	0.2425	370.4	3	559.55
3	1.2	480	4	19	20	4.8914	1.4197	3.3134	0.3299	480	4	480.43
4	1.5	250	3	3	17	4.2903	1.5085	2.564	0.6581	250	2.67	213.11
5	1.5	370.4	4	3	20	4.47	1.6009	2.68	0.6414	370.4	2.68	214.07
6	1.5	480	2	3	20	5.0415	1.6275	2.9224	0.4356	480	2	218.36
7	2	250	4	3	8	3.8796	1.4666	2.9054	0.8356	362.68	1.81	185.7
8	2	370.4	2	3	8	3.8923	1.4682	2.9135	0.8344	370.4	1.81	185.7
9	2	480	3	4	10	3.5682	1.5591	2.9968	1.0829	480	1.9	186.22

Table 18. Output of the stepwise regression analysis for the statistical sensitivity of n_1 to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	F	p-value
Regression	300.7086	1	300.7086	10.4458	0.0144
Residual	201.5136	7	28.78766		
Total	502.2222	8			

Table of regression coefficients

Parameters	Coefficient	Standard error	t	p-value
Constant	35.8878	8.6775	4.1357	0.0044
γ	-17.517	5.4199	-3.2320	0.0144

Table 19. Output of the stepwise regression analysis for the statistical sensitivity of n_2 to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	F	p-value
Regression	215.5283	1	215.5283	51.3852	0.0002
Residual	29.36054	7	4.1944		
Total	244.8889	8			

Table of regression coefficients

Parameters	Coefficient	Standard error	t	p-value
Constant	39.12245	3.3122	11.8115	7.07E-06
γ	-14.8299	2.0688	-7.1684	0.0002

Table 20. Output of the stepwise regression analysis for the statistical sensitivity of k_1 to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	F	p-value
Regression	2.1983	1	2.1983	38.577	0.0004
Residual	0.3989	7	0.0570		
Total	2.5972	8			

Table of regression coefficients

Parameters	Coefficient	Standard error	t	p-value
Constant	6.7950	0.3861	17.600	4.71E-07
γ	-1.4977	0.2411	-6.2110	0.0004

Table 21. Output of the stepwise regression analysis for the statistical sensitivity of *h* to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	*F*	*p*-value
Regression	0.6670	1	0.6670	47.1379	0.0002
Residual	0.0990	7	0.0142		
Total	0.7660	8			

Table of regression coefficients

Parameters	Coefficient	Standard error	*t*	*p*-value
Constant	-0.7124	0.1924	-3.7029	0.0076
γ	0.8250	0.1202	6.8657	0.0002

Table 22 shows that ATS_L significantly affects ATS_0, in which the value of ATS_0 increases when the value of ATS_L increases. On the other hand, Table 23 shows that the input parameters γ and ATS_U significantly affect the output parameter ATS_1, in which the value of ATS_1 increases when the value of ATS_U increases; whereas the value of ATS_1 decreases when the value of γ increases.

Table 22. Output of the stepwise regression analysis for the statistical sensitivity of ATS_0 to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	*F*	*p*-value
Regression	55267.56	1	55267.56	42.4914	0.0003
Residual	9104.739	7	1300.677		
Total	64372.3	8			

Table of regression coefficients

Parameters	Coefficient	Standard error	*t*	*p*-value
Constant	73.3129	48.4589	1.5129	0.1741
ATS_L	0.83426	0.1280	6.5185	0.0003

Table 23. Output of the stepwise regression analysis for the statistical sensitivity of ATS_1 to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	*F*	*p*-value
Regression	3.1914	2	1.5957	8.4127	0.0182
Residual	1.1380	6	0.1897		
Total	4.3294	8			

Table of regression coefficients

Parameters	Coefficient	Standard error	*t*	*p*-value
Constant	3.3249	0.8835	3.7632	0.0094
γ	-1.4265	0.4399	-3.2426	0.0176
ATS_U	0.4467	0.1778	2.5122	0.0458

Table 24. Output of the stepwise regression analysis for the statistical sensitivity of C to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	F	p-value
Regression	204614.7467	1	204614.7467	11.2801	0.0121
Residual	126976.2373	7	18139.4625		
Total	331590.9840	8			

Table of regression coefficients

Parameters	Coefficient	Standard error	t	p-value
Constant	1045.4287	217.8220	4.7995	0.0020
γ	-456.9361	136.0502	-3.3586	0.0121

Table 25. Output of the stepwise regression analysis for the statistical sensitivity of w to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	F	p-value
Regression	0.0187	3	0.0062	0.7249	0.5792
Residual	0.0430	5	0.0086		
Total	0.0617	8			

Table 26. Output of the stepwise regression analysis for the statistical sensitivity of k_2 to changes in the input parameters.

ANOVA table

Source	Sum of squares	Degree of freedom	Mean square	F	p-value
Regression	0.1934	3	0.0645	0.7258	0.5788
Residual	0.4442	5	0.0888		
Total	0.6376	8			

The underlying assumption of the statistical sensitivity analysis on C is satisfied, as shown in Figure 3, where the points plotted reasonably close to a straight line.

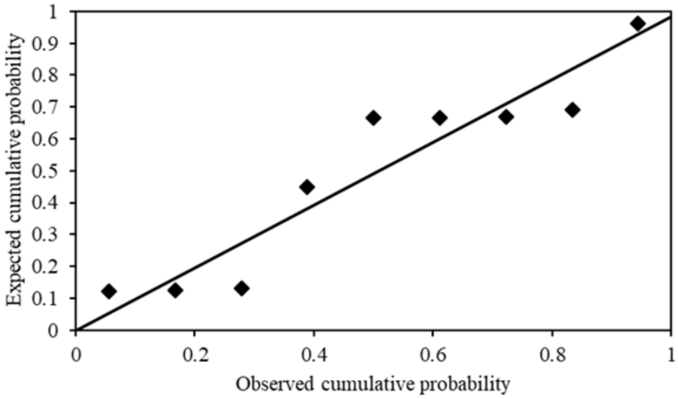

Figure 3. Normal probability plot of standardized residual for the statistical sensitivity of C to changes in input parameters

Furthermore, the factor's responses for the statistical sensitivity analysis on C are shown in Table 27, with the ranking of the parameter as γ, ATS_U, and ATS_L (from the highest to the lowest C) and this result is confirmed by the regression analysis, where C is only significantly affected by γ.

Table 27. Factor responses for the statistical sensitivity analysis on C.

Parameters		γ	ARL_L	ARL_U
Level	1	587.63	373.91	375.66
	2	215.18	319.77	319.63
	3	185.87	295.01	293.4
Effect		401.76	78.9	82.26
Ranking		1	3	2

5. Conclusions

In this study, an economic-statistical design of the DS s chart is investigated and its cost function is minimized to obtain the optimal design parameters subject to statistical constraints. The employment of the economic-statistical design for the DS s chart instead of the Shewhart s chart brings a significant saving in the cost. Note that the economic-statistical design of the DS s chart has a small increase in C but with a better statistical performance when it is used in place of the economic design.

1) A numerical example is presented and an economic sensitivity analysis is then performed to show the effects of cost and process parameters on the design of the economic-statistical design of the DS s chart, where the input parameter are λ, C_0, C_1, W, a, b, Y, T_0, T_1, T_2 and e. From the results of the economic sensitivity analysis, the following conclusions can be made.
2) n_2 is significantly affected by the parameter W only, in which a larger value of W results in a larger value of n_2.
3) w is significantly affected by the parameters e, C_1, λ, a and b, in which a larger value of e, C_1 or λ leads to a larger value in w, whereas a larger value of a or b leads to a smaller value in w.
4) k_1 is significantly affected by the parameters λ and C_1, in which a larger value of λ or C_1 leads to a larger value in k_1.
5) k_2 is significantly affected by the parameters λ, C_1 and b, in which a larger value of λ or C_1 leads to a larger value in k_2, whereas a larger value in b leads to a smaller value in k_2.
6) h is significantly affected by parameters b, C_1, e and λ, in which a larger value of b leads to a longer value in h, whereas a larger value of C_1, e or λ leads to a shorter value in h.
7) ATS_0 is significantly affected by parameters Y, λ, b and C_1, in which a larger value of Y leads to a longer ATS_0, whereas a larger value of λ, b or C_1 leads to a shorter ATS_0.
8) ATS_1 is significantly affected by parameters b, C_0, C_1 and λ, in which a larger value of b or C_0 leads to a longer ATS_1, whereas a larger value of C_1 and λ leads to a shorter ATS_1.
9) C is significantly affected by the parameters λ, C_0, C_1, b, W and e, in which a higher value in any of these parameters leads to a higher value in C.
10) n_1 is not significantly affected by any of the input parameters.

Furthermore, a statistical sensitivity analysis is also performed to show the effect of statistical constraints and shift on the economic-statistical design of the DS *s* chart, where the input parameters are γ, ATS_L and ATS_U. From the results of the statistical sensitivity analysis, the following conclusions can be made.

1) n_1, n_2, k_1 and h are significantly affected by parameter γ, in which a larger value of γ leads to a longer h, whereas a smaller value of n_1, n_2 or k_1 leads to a shorter h.
2) ATS_0 is significantly affected by parameter ATS_L, in which a larger value of ATS_L leads to a longer ATS_0.
3) ATS_1 is significantly affected by parameters γ and ATS_U, in which a larger value of ATS_U leads to a longer ATS_1, whereas a larger value in γ leads to a shorter ATS_1.
4) C is significantly affected by parameter γ, in which a larger magnitude of γ in the process tends to generate a lower value in C.
5) w and k_2 are not significantly affected by any of the input parameters.

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Appendix:

Table 4. The optimal solutions for the economic-statistical designs of the DS *s* chart and the Shewhart *s* chart

Case	DS *s* chart									Shewhart *s* chart						Percentage of decrease in *C*
	n_1	n_2	w	k_1	k_2	h	ATS_0	ATS_1	C_{DS}	n	h	k	ATS_0	ATS_1	C_S	
1	5	20	3.9124	1.2918	2.6048	1.2060	370.40	3.00	82.31	18	1.5418	2.7977	370.41	3.00	93.35	11.83
2	3	20	4.4417	1.4978	2.6619	0.7187	370.40	2.82	216.93	14	1.1443	2.9343	370.40	3.00	242.97	10.72
3	3	19	4.3618	1.7281	2.7525	0.5343	370.40	2.50	531.31	14	1.1442	2.9343	370.41	3.00	602.16	11.77
4	4	19	4.1443	1.4256	2.6304	0.9586	370.40	3.00	102.35	17	1.4479	2.8265	370.42	3.00	127.08	19.46
5	4	20	4.2273	1.2982	2.6105	1.0426	370.40	3.00	251.55	18	1.5418	2.7977	370.41	3.00	303.49	17.11
6	3	20	4.9250	1.9389	3.0734	0.1911	370.40	1.11	406.00	13	0.5086	3.2399	370.40	1.78	456.9	11.14
7	4	20	4.0643	1.4403	2.6165	0.9716	370.40	3.00	124.29	18	1.5418	2.7977	370.41	3.00	149.2	16.70
8	3	18	4.3917	1.6257	2.6762	0.6820	370.40	3.00	287.47	15	1.2491	2.8942	370.40	3.00	354.14	18.83
9	9	19	3.2743	1.1920	2.6698	1.6666	370.40	2.92	300.58	20	1.7123	2.7492	370.43	2.99	309.77	2.97
10	4	19	4.2030	1.4148	2.6257	0.9646	370.40	3.00	172.94	18	1.5418	2.7977	370.40	3.00	230.85	25.09
11	6	20	3.8595	1.2717	2.7628	1.3048	518.94	3.00	180.77	19	1.6312	2.7717	370.40	3.00	191.33	5.52
12	3	19	4.4326	1.7159	2.8046	0.4730	370.40	2.24	360.20	13	1.0364	2.9796	370.40	3.00	395.76	8.99
13	15	16	3.0761	1.3384	2.6235	2.0995	370.40	3.00	197.35	16	1.3506	2.8584	370.40	3.00	216.05	8.66
14	8	19	3.5712	1.3076	2.5868	1.5731	370.40	3.00	302.39	16	1.3505	2.8585	370.44	3.00	353.05	14.35
15	2	20	5.6382	1.9849	2.9591	0.1933	370.40	1.52	250.43	10	0.5030	3.2806	370.40	2.38	292.22	14.30
16	3	20	4.6598	1.6318	2.8070	0.4735	381.94	2.10	184.45	14	1.1443	2.9343	370.40	3.00	201.73	8.57
17	3	20	4.5013	1.4737	2.6255	0.7821	370.40	3.00	277.54	14	1.1442	2.9343	370.42	3.00	302.44	8.23
18	4	20	4.1558	1.3006	2.6231	1.0248	370.40	2.96	257.99	17	1.4482	2.8264	370.40	3.00	304.92	15.39
19	4	19	4.5293	1.3064	2.6179	1.0174	370.40	3.00	207.32	17	1.4482	2.8264	370.40	3.00	261.95	20.86
20	3	17	4.5783	1.7861	2.8551	0.4143	370.40	2.17	274.68	12	0.9262	3.0310	370.40	3.00	295.32	6.99
21	3	20	4.9305	1.5968	2.9569	0.3191	370.40	1.46	277.74	18	1.0785	2.9323	370.40	2.25	327.16	15.11
22	3	20	4.2638	1.4409	2.6535	0.7913	370.40	3.00	180.46	14	1.1443	2.9343	370.40	3.00	194.06	7.01
23	4	20	4.2568	1.4495	2.7603	0.6809	370.40	2.20	310.54	18	1.5418	2.7977	370.40	3.00	337.61	8.02
24	3	20	4.4327	1.4210	2.6344	0.8033	370.40	3.00	226.12	18	1.5418	2.7977	370.40	3.00	271.9	16.84
25	5	20	4.0125	1.3533	2.5893	1.1775	370.40	3.00	295.59	18	1.5418	2.7977	370.40	3.00	351.32	15.86
26	3	19	4.5466	1.7686	2.8926	0.3662	370.40	1.84	183.91	11	0.6902	3.1514	370.40	2.67	214.27	14.17
27	4	20	3.9952	1.3128	2.6348	1.0303	370.40	3.00	251.42	16	1.3506	2.8584	370.40	3.00	271.76	7.48
28	4	20	4.1421	1.2623	2.6358	1.0229	370.40	2.91	246.37	19	1.6312	2.7717	370.40	3.00	257	4.14
29	4	20	4.0931	1.5013	2.7617	0.6880	370.40	2.28	203.24	16	1.3506	2.8584	370.40	3.00	233.19	12.84
30	11	18	3.1147	1.4593	2.6694	1.7650	370.40	3.00	272.69	16	1.3506	2.8584	370.40	3.00	310.29	12.12
31	6	19	3.8471	1.2945	2.6012	1.3275	370.40	3.00	317.06	17	1.4482	2.8265	370.43	3.00	372.2	14.81
32	5	20	3.7862	1.2873	2.7033	1.1798	424.29	3.00	107.65	17	1.4482	2.8264	370.41	3.00	118.65	9.27
33	2	20	5.4496	1.9540	2.8259	0.2760	370.40	2.03	312.37	12	0.9250	3.0315	370.40	3.00	357.13	12.53
34	5	20	3.9443	1.3035	2.6002	1.2010	370.40	3.00	254.62	17	1.4463	2.8289	372.29	3.00	277.99	8.41
35	5	19	3.9464	1.3122	2.6199	1.1719	370.40	3.00	192.99	17	1.4482	2.8264	370.40	3.00	246.32	21.65
36	2	20	5.9946	1.8470	3.0782	0.1538	370.40	1.17	292.34	13	0.4898	3.2533	370.40	1.73	345.87	15.48
Average																12.59

Table 5. The optimal solutions for the economic-statistical and economic designs of the DS s chart.

Case	Economic-statistical design									Economic design									Percentage of increase in ATS_0	Percentage of increase in ATS_1	Percentage of increase in C
	n_1	n_2	w	k_1	k_2	h	ATS_0	ATS_1	C	n_1	n_2	w	k_1	k_2	h	ATS_0	ATS_1	C			
1	5	20	3.9124	1.2918	2.6048	1.206	370.4	3	82.31	6	18	1.1453	3.2951	2.369	1.8169	247.55	3.71	81.73	49.64	-19.14	0.71
2	3	20	4.4417	1.4978	2.6619	0.7187	370.4	2.82	216.93	4	14	1.2525	3.4339	2.3064	0.9699	132.23	2.78	214.76	180.12	1.44	1.01
3	3	19	4.3618	1.7281	2.7525	0.5343	370.4	2.5	531.31	3	9	1.2602	3.1127	2.0413	0.6569	47.85	2.35	509.12	674.09	6.38	4.36
4	4	19	4.1443	1.4256	2.6304	0.9586	370.4	3	102.35	4	15	1.2656	3.4554	2.3173	1.8607	268.49	5.28	96.59	37.96	-43.18	5.96
5	4	20	4.2273	1.2982	2.6105	1.0426	370.4	3	251.55	6	15	0.9688	2.8741	2.0941	2.4016	152.39	4.51	241.07	143.04	-33.48	4.35
6	3	20	4.925	1.9389	3.0734	0.1911	370.4	1.11	406	3	11	1.5508	3.5604	2.3136	0.2352	38.6	1.02	394.15	859.83	8.82	3.01
7	4	20	4.0643	1.4403	2.6165	0.9716	370.4	3	124.29	4	18	1.3637	3.8939	2.5355	1.2149	342.79	3.65	123.39	8.06	-17.81	0.73
8	3	18	4.3917	1.6257	2.6762	0.682	370.4	3	287.47	4	5	0.8539	1.8959	1.4638	1.3884	20.96	3.1	261.53	1667.18	-3.23	9.92
9	9	19	3.2743	1.192	2.6698	1.6666	370.4	2.92	300.58	8	18	1.1025	3.0795	2.479	1.6173	221.33	2.85	300.13	67.36	2.46	0.15
10	4	19	4.203	1.4148	2.6257	0.9646	370.4	3	172.94	5	11	0.9888	2.6904	1.9158	2.0384	89.34	4.33	161.7	314.6	-30.72	6.95
11	6	20	3.8595	1.2717	2.7628	1.3048	518.94	3	180.77	6	20	1.2543	3.8249	2.75	1.4066	533.75	3.2	180.71	-2.77	-6.25	0.03
12	3	19	4.4326	1.7159	2.8046	0.473	370.4	2.24	360.2	4	7	1.1438	2.4284	1.8429	0.7749	26.72	2.08	342.46	1286.23	7.69	5.18
13	15	16	3.0761	1.3384	2.6235	2.0995	370.4	3	197.35	6	13	0.9122	2.6545	1.9616	1.704	76.12	3.13	187.91	386.6	-4.15	5.02
14	8	19	3.5712	1.3076	2.5868	1.5731	370.4	3	302.39	5	11	1.0977	2.7508	1.9669	3	152	6.71	274.95	169.26	-60.53	10.18
15	2	20	5.6382	1.9849	2.9591	0.1933	370.4	1.52	250.43	2	17	1.8625	5.0604	2.6761	0.2067	161.36	1.46	248.51	129.56	4.11	0.77
16	3	20	4.6598	1.6318	2.807	0.4735	381.94	2.1	184.45	3	20	1.6318	4.6598	2.807	0.4735	381.94	2.1	184.45	0	0	0
17	3	20	4.5013	1.4737	2.6255	0.7821	370.4	3	277.54	4	10	1.0222	2.8103	1.9466	1.9468	96.28	4.91	270.45	284.75	-38.9	2.62
18	4	20	4.1558	1.3006	2.6231	1.0248	370.4	2.96	257.99	6	10	0.8388	2.3378	1.7953	1.5912	45.22	2.87	240.49	719.11	3.14	7.28
19	4	19	4.5293	1.3064	2.6179	1.0174	370.4	3	207.32	4	8	1.0112	2.4843	1.6857	3	88.06	7.25	175.05	395.52	-65.64	19.13
20	3	17	4.5783	1.7861	2.8551	0.4143	370.4	2.17	274.68	3	15	1.6937	4.124	2.606	0.4419	184.98	2.1	273.85	100.24	3.33	0.3
21	3	20	4.9305	1.5968	2.9569	0.3191	370.4	1.46	277.74	4	15	1.3783	3.6556	2.4763	0.4427	96.77	1.39	271.76	282.76	5.04	2.2
22	3	20	4.2638	1.4409	2.6535	0.7913	370.4	3	180.46	4	15	1.3401	3.628	2.5083	1.0062	225.52	3.13	179.83	64.24	-4.15	0.35
23	4	20	4.2568	1.4495	2.7603	0.6809	370.4	2.2	310.54	4	18	1.3647	3.8689	2.5306	0.7174	199.2	2.15	309.59	85.95	2.33	0.31
24	3	20	4.4327	1.421	2.6344	0.8033	370.4	3	226.12	5	6	0.5842	1.7218	1.3514	2.4011	27.34	4.2	201.45	1254.3	-28.57	12.25
25	5	20	4.0125	1.3533	2.5893	1.1775	370.4	3	295.59	5	14	1.0352	3.0909	2.125	2.8674	219.18	6.24	277.46	69	-51.92	6.53
26	3	19	4.5466	1.7686	2.8926	0.3662	370.4	1.84	183.91	3	10	1.321	3.2593	2.1641	0.4589	45.03	1.72	174.18	722.56	6.98	5.59
27	4	20	3.9952	1.3128	2.6348	1.0303	370.4	3	251.42	6	15	1.0847	2.9943	2.2482	1.5517	138.51	3.14	249.41	167.42	-4.46	0.81
28	4	20	4.1421	1.2623	2.6358	1.0229	370.4	2.91	246.37	6	17	1.1294	3.199	2.3385	1.4435	174.37	2.95	245.61	112.41	-1.36	0.31
29	4	20	4.0931	1.5013	2.7617	0.688	370.4	2.28	203.24	4	13	1.2125	3.2902	2.2559	0.7826	88.54	2.21	197.92	318.34	3.17	2.69
30	11	18	3.1147	1.4593	2.6694	1.765	370.4	3	272.69	5	10	1.0724	2.6153	1.9904	2.0778	97.07	4.69	247.14	281.58	-36.03	10.34
31	6	19	3.8471	1.2945	2.6012	1.3275	370.4	3	317.06	6	13	0.9177	2.669	1.9218	2.595	111.09	4.74	304.27	233.42	-36.71	4.2
32	5	20	3.7862	1.2873	2.7033	1.1798	424.29	3	107.65	6	19	1.2902	3.5788	2.6897	1.3549	409.85	3.12	107.65	3.53	-3.85	0
33	2	20	5.4496	1.954	2.8259	0.276	370.4	2.03	312.37	3	8	1.356	2.8977	1.9622	0.4928	29.48	1.82	296.83	1156.45	11.54	5.24
34	5	20	3.9443	1.3035	2.6002	1.201	370.4	3	254.62	5	17	1.1796	3.4732	2.4488	3	523.85	7.15	244.16	-29.15	-58.16	4.28
35	5	19	3.9464	1.3122	2.6199	1.1719	370.4	3	192.99	6	8	0.681	1.9255	1.4801	2.4909	37.22	4.11	176.21	895.16	-27.01	9.52
36	2	20	5.9946	1.847	3.0782	0.1538	370.4	1.17	292.34	2	16	1.6612	4.8708	2.5333	0.1743	83.26	1.07	285.62	344.87	9.35	2.35
Average																			373.15	-13.87	4.3