International Journal for Quality Research 7(3) 3–14
ISSN 1800-6450

Ashit Chakraborty[1]
Anwer Khurshid

Article info:
Received 27 July 2013
Accepted 30 August 2013

UDC – 65.012.7

MEASUREMENT ERROR EFFECT ON THE POWER OF CONTROL CHART FOR ZERO-TRUNCATED POISSON DISTRIBUTION

Abstract: *Measurement error is the difference between the true value and the measured value of a quantity that exists in practice and may considerably affect the performance of control charts in some cases. Measurement error variability has uncertainty which can be from several sources. In this paper, we have studied the effect of these sources of variability on the power characteristics of control chart and obtained the values of average run length (ARL) for zero-truncated Poisson distribution (ZTPD). Expression of the power of control chart for variable sample size under standardized normal variate for ZTPD is also derived.*

Keywords: *Measurement error, zero truncated Poisson distribution (ZTPD), Average Run Length (ARL), power*

1. Introduction

Measurement is seldom, if ever, without error and is a significant issue in control chart. Often subject to measurement error, the process variability is observed in any control chart which is the combination of inherent variability in the processes and the error due to the measurement instrument. If the measurement error is large relative to the process variability, the control chart to detect any shift in the process level is affected (Kanazuka, 1986). For a discussion on the measurement error and its effect on the performance on control charts (Ryan, 2011).

The consequences of measurement error on the actual performance of various control charts have long been a concern and studied by several authors. The effect of measurement errors for $\overline{X}$ chart was discussed (Bennett, 1954; Mizuno, 1961; Abraham, 1977; Mittag and Stemann, 1998). Singh (1964) considered measurement error in acceptance sampling for attributes. Kanazuka (1986) and Mittag (1995) studied the effect of measurement error on the power of the $\overline{X}-R$ control charts. Rahim (1985) observed the effect of non-normality and measurement errors on the economic design of charts. Walden (1990) measured the power of $\overline{X}$, R and $\overline{X}-R$ charts using ARL when measurement error affects the system. Linna (1991) studied the effect of increasing the measurement variance and slope of covariate model on Shewhart control charts. Tricker *et al.* (1998) investigated the effects of one particular aspect of measurement error (round-off) on R control chart.

Moreover, (Linna and Woodall, 2001; Linna *et al.*, 2001) studied the effect of measurement error on Shewhart control charts using a linear covariate and multivariate control charts respectively.

[1] Corresponding author: Ashit B. Chakraborty
email: abc_sac@rediffmail.com

Stemann and Weihs (2001) and Maravelakis *et al*. (2004) investigated the effect of measurement error on the EWMA chart. Shore (2004) derived the requirements of measurement error, to satisfy the various control charts. Yang (2002) investigated the effect of measurement error on the asymmetric economic design and S control charts. Chang and Gan (2006) developed Shewhart chart for monitoring the linearity between two measurement gauges. Huwang and Hung (2007) considered the effect of measurement error on the control charts for monitoring multivariate process variability. Yang *et al*. (2007) derived a process model to take into account of measurement error on two dependent processes (Yang and Yang, 2005). Xiaohong and Zhaojun (2009) investigated the effect of measurement error on the CUSUM chart for the autoregressive data. Costa and Castagliola (2011) examined the effect of measurement error and autocorrelation on the $\overline{X}$ chart. Moameni *et al*. (2012) studied the effect of measurement error on the effectiveness of the fuzzy control chart to detect out of control situations. Maravelakis (2012) considered the old problem and investigated the effect of measurement error on the performance of the CUSUM control chart for the mean. More recently, Yang *et al*. (2013) proposed a new EWMA control chart to monitor the exponentially distributed service time between consecutive events with the measurement error instead of monitoring the number of events in a given time interval.

The purpose of this paper is to study the effect of the two sources of variability on the power characteristics of control chart and to obtain the values of average run length (ARL) for zero-truncated Poisson distribution (ZTPD). Expression of the power of control chart for variable sample size under standardized normal variate for ZTPD is also derived. Effects of measurement error on control charts for the ratio of two Poisson distributions, as studied by (Sahai and Khurshid, 1993) is dealt in a separate paper (Chakraborty and Khurshid, 2013).

2. Power of control chart for ZTPD in the presence of measurement error

2.1 Zero truncated Poisson distribution

A probability distribution can be classified into four types, left, right, double and multiple truncation. The most common form of left truncation is the exclusion of the zero class. Probability distributions often arise in practice which are of the Poisson type, but in which the zero value is unobserved. This may occur in the situations when the observational apparatus becomes active when at least one event occurs. Examples of ZTPD may be found in many areas, such as, the number of accidents per workers in a factory, the number of persons per house suffering from an infectious disease or number of surface defects in x-ray film etc. A zero-truncated Poisson or positive Poisson random variable (Johnson *et al*., 2005) also called conditional Poisson random variable (Cohen, 1960) is a Poisson distribution with parameter λ and $p(0)=0.$ Thus, it is necessitated to scale the other probabilities by a factor of $\dfrac{1}{1-p(0)}$ where $p(0)=e^{-\lambda}$, the original probability that $x=0$, in order to still have a discrete probability function. Let $x_1, x_2, \ldots, x_n$ be independent random variables each having a ZTPD with probability function

$$f(X=x;\lambda)=\frac{e^{-\lambda}\lambda^{x}}{x!(1-e^{-\lambda})} \qquad (2.1)$$

for $x=1,2,\ldots,$ where $\lambda>0.$ The mean and variance of the above function are

$$\mu = \frac{\lambda}{(1-e^{-\lambda})}, \qquad (2.2)$$

$$\sigma^2 = \frac{\lambda\,[1-e^{-\lambda}(1+\lambda)]}{(1-e^{-\lambda})^2}. \qquad (2.3)$$

The cumulative sum (CUSUM) and Shewhart control charts for ZTPD were developed by (Chakraborty and Kakoty, 1987; Chakraborty and Singh, 1990) respectively. More recently, Balamurali and Kalyanasundaram (2013) have developed design and implementation procedures of CUSUM control schemes based on zero truncated Poisson distribution.

2.2 Assumptions and notations

In the development of the power of the control chart and ARL for equation (2.1), the following assumptions are made and notations are used:

1. The process has ZTPD with mean $\mu = \lambda_p[(1-e^{-\lambda_p})]^{-1}$ and variance $\sigma_p^2 = (1-e^{-\lambda_p})^{-2}\,\{\lambda_p\,[1-e^{-\lambda_p}(1+\lambda_p)]\}$ where σ_p^2 denotes process (inherent) variability;
2. The measurement process has a variance σ_m^2. Thus, $\sigma^2 = \sigma_p^2 + \sigma_m^2$;
3. The process is in a state of statistical control at the time of determining the control limits and the same measuring instrument is used for later measurements;
4. When the process parameter shifts, the data is also come from ZTPD with mean μ' and variance $\sigma_p'^2 + \sigma_m^2$; and
5. The measurement of items have been taken to ascertain the number of defects per unit.

Under the above assumptions, Shewhart control limits for c chart will be $\mu \pm K\sqrt{\sigma_p^2+\sigma_m^2}$. Usually K is taken as 3 for the calculation of control limits (Montgomery, 2013) as it covers at least 99.73% of samples which is based on Shewhart's claim that control limits at 3 standard errors are the most economical (Wheeler and Chambers, 2010). Hence the control limits are known as 3σ limits.

If we assume that X is a Poisson variate with mean μ and variance $\sigma^2 = \sigma_p^2 + \sigma_m^2$, then following (Kanazuka, 1986), the power of detecting the change of process parameter for c chart is given by

$$P_C = P\left\{X \ge \mu + K\sqrt{\sigma_p^2+\sigma_m^2}\right\} + P\left\{X \le \mu - K\sqrt{\sigma_p^2+\sigma_m^2}\right\}. \qquad (2.4)$$

Thus, using equations (2.2) and (2.3), we have

$$P_C = P\left\{ X \ge \frac{\lambda_p}{(1-e^{-\lambda_p})} + K\sqrt{\frac{\lambda_p\{1-e^{-\lambda_p}(1+\lambda_p)}{(1-e^{-\lambda_p})^2} + \frac{\lambda_m\{1-e^{-\lambda_m}(1+\lambda_m)\}}{(1-e^{-\lambda_m})^2}} \right\}$$

$$+P\left\{ X \le \frac{\lambda_p}{(1-e^{-\lambda_p})} - K\sqrt{\frac{\lambda_p\{1-e^{-\lambda_p}(1+\lambda_p)}{(1-e^{-\lambda_p})^2} + \frac{\lambda_m\{1-e^{-\lambda_m}(1+\lambda_m)\}}{(1-e^{-\lambda_m})^2}} \right\}$$

$$= \left[1 - P\left\{X \le UCL\right\}\right] + P\left\{X \le LCL\right\}$$

where λ_m is the process parameter of ZTPD, when the measurement process has a variance σ_m^2.

Hence, following equation (2.1), we have

$$P_C = P\left[1 - \sum_{x=1}^{UCL} \frac{e^{-\lambda}\lambda^x}{x!\,(1-e^{-\lambda})}\right] + P\left[\sum_{x=1}^{LCL} \frac{e^{-\lambda}\lambda^x}{x!\,(1-e^{-\lambda})}\right]. \quad (2.5)$$

The calculation of P_C for equation (2.5) is shown below in Table 1.

Table 1. Power of control chart for ZTPD (under measurement error) $\lambda_p = 2$, $\sigma_p^2 = 1.5887$

			$\lambda_m = 0.2$, $\sigma_m^2 = 0.1064$, $UCL = 6.21$		$\lambda_m = 0.9$, $\sigma_m^2 = 0.5815$, $UCL = 7$		$\lambda_m = 1.5$, $\sigma_m^2 = 1.5$, $UCL = 8$	
λ'_p	$d = (\lambda'_p - \lambda_p)$	σ'^2_p	P_C	$R^2 = \sigma_m^2/\sigma'^2_p$	P_C	$R^2 = \sigma_m^2/\sigma'^2_p$	P_C	$R^2 = \sigma_m^2/\sigma'^2_p$
-	-	-	0.0052	0.0670*	0.0013	0.3660*	0.0002	0.9442*
3	1	2.6611	0.0353	0.0400	0.0125	0.2185	0.0004	0.5637
4	2	3.7705	0.1128	0.0282	0.0521	0.1542	0.0218	0.3978
5	3	4.8632	0.2394	0.0219	0.1343	0.1196	0.0656	0.3084
6	4	5.9252	0.3947	0.0180	0.2566	0.0981	0.1532	0.2532

* $(=\sigma_m^2/\sigma_p^2)$

It can be seen from Table 1 that:

1. increase in the shift of process parameter from λ_p to λ'_p, there is an increase in the power of the control chart P_C for fixed λ_m, σ_m^2 and UCL. Smaller the deviation $d = (\lambda'_p - \lambda_p)$, smaller the power of the test;
2. relative measurement error (R^2) tends to decrease as the power of control chart increase, resulting in increases in the shift of the process parameter (for fixed λ_m, σ_m^2 and UCL); and
3. for fixed deviation, the values of P_C decrease and R^2 increase as the values of λ_m, σ_m^2 and UCL are increased.

3. Power of control chart for (for variable sample size) under standardization procedure

Instead of plotting the number of defects in the control chart, we can standardize the variates which can be plotted accordingly.

This stabilizes the variables and the resulting control chart. In this case, the control limits as well as central lines are invariant with sample size n.

Thus, equation (2.4) can be expressed in terms of standardized normal variable Z (when sample size is large and varies)

$$Z = \frac{\bar{x} - \mu}{\sqrt{(\sigma_p^2 + \sigma_m^2)/n}} \qquad (3.1)$$

where $\mu = \lambda_p (1 - e^{-\lambda_p})^{-1}$,

$$\sigma_p^2 = \frac{\lambda_p \{1 - e^{-\lambda_p}(1 + \lambda_p)\}}{(1 - e^{-\lambda_p})^2}, \quad \text{and}$$

$$\sigma_m^2 = \frac{\lambda_m \{1 - e^{-\lambda_m}(1 + \lambda_m)\}}{(1 - e^{-\lambda_m})^2}.$$

Hence, following (Kanazuka, 1986) and using equation (3.1), when the process parameter changes from μ to μ', the power of the control chart for ZTPD is

$$P_C = P\left\{ \frac{\bar{x} - \mu'}{\sqrt{(\sigma_p'^2 + \sigma_m^2)/n}} \geq \frac{(\mu - \mu')\sqrt{n}}{\sqrt{\sigma_p'^2 + \sigma_m^2}} + 3\frac{\sqrt{\sigma_p^2 + \sigma_m^2}}{\sqrt{\sigma_p'^2 + \sigma_m^2}} \right\}$$

$$+ P\left\{ \frac{\bar{x} - \mu'}{\sqrt{(\sigma_p'^2 + \sigma_m^2)/n}} \leq \frac{(\mu - \mu')\sqrt{n}}{\sqrt{\sigma_p'^2 + \sigma_m^2}} - 3\frac{\sqrt{\sigma_p^2 + \sigma_m^2}}{\sqrt{\sigma_p'^2 + \sigma_m^2}} \right\}$$

$$= P\left\{ Z \geq \frac{((\mu - \mu')/\sigma_p)\sqrt{n}}{\sqrt{(\sigma_p'^2/\sigma_p^2) + (\sigma_m^2/\sigma_p^2)}} + 3\frac{\sqrt{1 + (\sigma_m^2/\sigma_p^2)}}{\sqrt{(\sigma_p'^2/\sigma_p^2) + (\sigma_m^2/\sigma_p^2)}} \right\}$$

$$+ P\left\{ Z \leq \frac{((\mu - \mu')/\sigma_p)\sqrt{n}}{\sqrt{(\sigma_p'^2/\sigma_p^2) + (\sigma_m^2/\sigma_p^2)}} - 3\frac{\sqrt{1 + (\sigma_m^2/\sigma_p^2)}}{\sqrt{(\sigma_p'^2/\sigma_p^2) + (\sigma_m^2/\sigma_p^2)}} \right\}$$

$$= P\left\{ Z \geq \frac{d_{\lambda_p}\sqrt{n}}{\sqrt{k_{\lambda_p}^2 + R^2}} + 3\frac{\sqrt{1 + R^2}}{\sqrt{k_{\lambda_p}^2 + R^2}} \right\} + P\left\{ Z \leq \frac{d_{\lambda_p}\sqrt{n}}{\sqrt{k_{\lambda_p}^2 + R^2}} - 3\frac{\sqrt{1 + R^2}}{\sqrt{k_{\lambda_p}^2 + R^2}} \right\} \qquad (3.2)$$

where $d_{\lambda_p} = \{(\mu - \mu')/\sigma_p)\}$, $k_{\lambda_p}^2 = \sigma_p'^2/\sigma_p^2$ and $R^2 = \sigma_m^2/\sigma_p^2$.

4. Average Run Length (ARL) under measurement error

To study the sensitivity of the monitoring procedure both the average run length (ARL) and operating characteristic function are examined. ARL is the average number of points that must be plotted before a point indicates an out of control condition. For any Shewhart control chart, the ARL is

$ARL = [P]^{-1}$ where P is the probability that a single point exceeds the control limits. Now, if the mean shifts from the incontrol value μ_0 to $\mu_1 = \mu_0 + k\sigma$, the probability of not detecting this shift on the first subsequent sample or the (β risk)

(Montgomery, 2013) is

$$\beta = P\left\{X \le UCL/\lambda_p\right\} - P\left\{X \le LCL/\lambda_p\right\} \quad (4.1)$$

Hence, $P = 1-\beta$ and

$$ARL = [1-\beta]^{-1} \quad (4.2)$$

The operating characteristic (OC) function expressed by the type II error probability β, is a measure of the inability of the control chart to detect the process shifts can be constructed for ZTPD by plotting β risk against the magnitude of the shift of the process parameter that is to be detected. The larger the β, the higher the probability that a control chart fails to detect the shift and vice-versa.

Thus, for ZTPD, equation (4.1), under measurement error becomes

$$\beta = P\left\{ X \le \frac{\lambda_p}{(1-e^{-\lambda_p})} + 3\left[\frac{\lambda_p\{1-e^{-\lambda_p}(1+\lambda_p)}{(1-e^{-\lambda_p})^2} + \frac{\lambda_m\{1-e^{-\lambda_m}(1+\lambda_m)\}}{(1-e^{-\lambda_m})^2}\right]^{-1}\right\}$$
$$- P\left\{ X \le \frac{\lambda_p}{(1-e^{-\lambda_p})} - 3\left[\frac{\lambda_p\{1-e^{-\lambda_p}(1+\lambda_p)}{(1-e^{-\lambda_p})^2} + \frac{\lambda_m\{1-e^{-\lambda_m}(1+\lambda_m)\}}{(1-e^{-\lambda_m})^2}\right]^{-1}\right\}. \quad (4.3)$$

Hence, substituting equation (4.3) in equation (4.2), we can obtain ARL. The values of β and ARL are shown in Table 2.

Table 2. Values of β and ARL for ZTPD control chart (under measurement error) $\lambda_p = 2$, $\sigma_p^2 = 1.5887$

	$\lambda_m = 0.2$, $\sigma_m^2 = 0.1064$, $UCL = 6.21$		$\lambda_m = 0.9$, $\sigma_m^2 = 0.5815$, $UCL = 7$		$\lambda_m = 1.5$, $\sigma_m^2 = 1.5$, $UCL = 8$	
λ'_p	β	ARL	β	ARL	β	ARL
-	0.9948	192.31	0.9987	769.23	0.9998	5000.00
3	0.9647	28.33	0.9875	80.00	0.9996	2500.00
4	0.8872	8.87	0.9475	19.19	0.9782	45.87
5	0.7606	4.18	0.8657	7.44	0.9344	15.24
6	0.6053	2.53	0.7434	3.90	0.8468	6.53

It is observed from the Table 2 that the values of ARL tend to decrease as the shift of the process parameters increase for fixed λ_m, σ_p^2, σ_m^2 and UCL. Whereas for fixed deviation, ARL values tend to increase as the values of λ_m, σ_m^2 and UCL tend to increase. Further, if there is an increase in σ_m^2, keeping σ_p^2 fixed, that β value increases as there is a decrease in deviation. Higher β values may become a matter of concern since accepting a null hypothesis $H_0 : \lambda = \lambda_p$ when it is false involves cost. Thus, where it is necessary to have a sample of small size, P_C should be set at a relatively high level so that the resultant β value does not become a matter of excessive concern.

5. Conclusion

We have drawn following conclusions from Tables 1 and 2:

1. When plotted, values of P_C so computed for different values of λ'_p, given that $\lambda'_p > \lambda_p$ yield power for the control chart. It shows that for $H_0 : \lambda = \lambda_p$ and $H_1 : \lambda'_p > \lambda_p$, the ideal situation is one in which $1-\beta = 0$ when $\lambda = \lambda_p$ and $1-\beta = 1$ when $\lambda'_p > \lambda_p$. However, the ideal situation can never be achieved because P_C and β are always in opposite direction. The only way to move towards the ideal on both sides is to increase the sample size, keeping P_C as fixed.
2. A reduction in P_C leads to an increase in β. In other words, a reduction in P_C is possible at the cost of an increase in β.
3. The reduction in the acceptance region (β) shifts the power curve upward.
4. Increase in sample size n from n_1 to n_2 shift the power curve upward.
5. Values of ARL and β tend to decrease as the relative measures R^2 also tend to decrease as the process parameter λ_p to λ'_p increase.

Further β, which is also considered as consumer risk, in the sense that one has to accept certain percentage of considerably bad lots or products. To protect oneself against poor quality, the consumer usually demands a small value of β for incoming quality P. From the Table 1 it has been observed the values of β changes as we increase the value of σ_m^2. This implies that the consumer will be affected if there is any measurement error in the product.

References:

Abraham, B. (1977). *Control charts and measurement error*. Milwaukee, WI: ASQC Technical Conference Transactions. American Society for Quality Control, 370-374.

Balamurali, S., & Kalyanasundaram, M. (2013). Design of cumulative sum control schemes for truncated Poisson distribution. *International Journal of Productivity and Quality Management, 12*, 94-119.

Bennet, C. A. (1954). Effect of measurement error on chemical process control. *Industrial Quality Control, 10*, 17-20.

Chakraborty, A. B., & Kakoty, S. K. (1987). Cumulative sum control charts for ZTPD. *Journal of Indian Association for Productivity, Quality and Reliability, 12*, 17-25.

Chakraborty, A. B., & Khurshid, A. (2013). Measurement error effect on the power of control chart for the ratio of two Poisson distributions. *Economic Quality Control (Accepted for publication)*.

Chakraborty, A. B., & Singh, B. P. (1990). *Shewhart control chart for ZTPD*. Trivandrum, India: Proceedings of National Seminar on Quality and Reliability, NIQR, 18-24.

Chang, T. C., & Gan, F. F. (2006). Montoring linearity of measurement gauguges. *Journal of Statistical Computation and Simulation*, *76*, 889-911.

Cohen, A. C. (1960). Estimating the parameter in a conditional Poisson distribution. *Biometrics*, *16*, 203-211.

Costa, A. F. B., & Castagliola, P. (2011). Effect of measurement error and autocorrelation on the $\overline{X}$ chart. *Journal of Applied Statistics*, *38*, 661-673.

Huwang, L., & Hung, Y. (2007). Effect of measurement error on monitoring multivariate process variability. *Statistica Sinica*, *17*, 749-760.

Johnson, N. L., Kemp, A. W. & Kotz, S. (2005). *Univariate Discrete Distributions,* Third Edition. New York: John Wiley and Sons.

Kanazuka, T. (1986). The effects of measurement error on the power of $\overline{X}-R$ charts. *Journal of Quality Technology*, *18*, 91-95.

Linna, K. W. (1991). *Control chart performance under linear covariate measurement processes*. Unpublished Ph. D. Thesis, University of Alabama, USA.

Linna, K. W., & Woodall, W. H. (2001). Effect of measurement error on Shewhart control charts. *Journal of Quality Technology*, *33*, 213-222.

Linna, K. W., Woodall, W. H., & Busby, K. L. (2001). The performances of multivariate effect control charts in presence of measurement error. *Journal of Quality Technology*, *33*, 349-355.

Maravelakis, P. E. (2012). Measurement error effect on the CUSUM control chart. *Journal of Applied Statistics*, *39*, 323-336.

Maravelakis, P. E., Panaretos, J., & Psarakis, S. (2004). EWMA chart and measurement error. *Journal of Applied Statistics*, *31*, 445-455.

Mittag, H. J. (1995). *Measurement error effect on control chart performance. ASQC Technical Conference Transactions*. Milwaukee, WI: American Society for Quality Control. 66-73.

Mittag, H. J., & Stemann, D. (1998). Gauge inspection effect on the performance of the $\overline{X}-S$ control chart. *Journal of Applied Statistics*, *25*, 307-317.

Mizuno, S. (1961). Problems on measurement errors in process control. *Bulletin of the International Statistical Institute*, *38*, 405-415.

Moameni, M., Saghaei, A., & Salanghooch, M. G. (2012). The effect of measurement error on $\overline{X}-R$ fuzzy control charts. *ETASR- Engineering, Technology and Applied Science Research*, *2*, 173-176.

Montgomery, D. C. (2013). *Introduction to Statistical Quality Control, Seventh Edition*. New York: John Wiley and Sons.

Rahim, M. A. (1985). Economic model of charts under non-normality and measurement error. *Computers and Operations Research*, *12*, 291-299.

Ryan, T. P. (2011). *Statistical Methods for Quality Improvement*, Third Edition. New York: John Wiley and Sons.

Sahai, H., & Khurshid, A. (1993). Confidence intervals for the ratio of two Poisson means. *The Mathematical Scientist*, *18*, 43-50.

Shore, H. (2004). Determining measurement error requirements to satisfy statistical process control performance requirements. *IIE Transactions*, *36*, 881-890.

Singh, H. R. (1964). Measurement error in acceptance sampling for attributes. *ISQC Bulletin*, *x*, 29-36.

Stemann, D., & Weihs, C. (2001). The EWMA-X-S control chart and its performance in the case of precise and imprecise data. *Statistical Papers*, *42*, 207-223.

Tricker, A., Coates, E., & Okell, E. (1998). The effect on the R chart of precision of measurement. *Journal of Quality Technology*, *30*, 232-239.

Walden C. T. (1990). *An analysis of variable control charts in the presence of measurement error*. Unpublished Master's Thesis, Department of Industrial Engineering, Mississippi State University, USA.

Wheeler, D. J., & Chambers, D. S. (2010). *Understanding Statistical Process Control*, Third Edition. Knoxville, TN: SPC Press.

Xiaohong, L., & Zhaojun, W. (2009). The CUSUM control chart for the autocorrelated data with measurement error. *Chinese Journal of Applied Probability*, *25*, 461-474.

Yang, S-F. (2002). The effects of imprecise measurement on the economic asymmetric and S control charts. *The Asian Journal on Quality*, *3*, 46-55.

Yang, S-F., & Yang, C-M. (2005). Effects of imprecise measurement on the two dependent processes control for the autocorrelated observations. *The International Journal of Advanced Manufacturing Technology*, *26*, 623-630.

Yang, S-F., Han-Wei, H., & Rahim, M. A. (2007). Effect of measurement error on controlling two dependent process steps. *Economic Quality Control*, *22*, 127-139.

Yang, S-F., Yang, C-M., & Cheng, G-F. (2013). *Effects of measurement error on process signal detection*. Applied Mechanics and Materials, 263-266, 496-500.

Ashit Chakraborty
St. Anthony's College,
Shillong,
Meghalaya,
India
abc_sac@rediffmail.com

Anwer Khurshid
University of Nizwa
College of Arts and
Sciences,
Department of Mathematical
and Physical Sciences,
Birkat Al Mouz,
Oman
anwer@unizwa.edu.om