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THE ROLE OF AUTOMATIC SHAPE AND POSITION RECOGNITION IN STREAMLINING MANUFACTURING

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Abstract: *The main features of most components consist of simple basic functional geometries: planes, cylinders, spheres and cones. Shape and position recognition of these geometries is essential for dimensional characterization of components, and represent an important contribution in the life cycle of the product, concerning in particular the manufacturing and inspection processes of the final product. This work aims to establish an algorithm to automatically recognize such geometries, without operator intervention. Using differential geometry large volumes of data can be treated and the basic functional geometries to be dealt recognized. The original data can be obtained by rapid acquisition methods, such as 3D survey or photography, and then converted into Cartesian coordinates. The satisfaction of intrinsic decision conditions allows different geometries to be fast identified, without operator intervention. Since inspection is generally a time consuming task, this method reduces operator intervention in the process. The algorithm was first tested using geometric data generated in MATLAB and then through a set of data points acquired by measuring with a coordinate measuring machine and a 3D scan on real physical surfaces. Comparison time spent in measuring is presented to show the advantage of the method. The results validated the suitability and potential of the algorithm hereby proposed.*

Keywords: *Shape recognition, Gaussian curvatures, flatness, sphericity, cylindricity, conicity, metrology*

1. Introduction

The actual geometric shape of any body is determined by the surfaces which delimit it. The surface geometry is defined by the design or manufacturing process, regardless of form deviations. When controlling manufactured parts it is important to

consider the effective surface, which is approximately depicted by a set of points actually taken from measures made on the surface of the part (ISO 1101). The main functional geometries of most mechanical manufactured components consist of some simple shapes, including the following basic ones: planes, cylinders, spheres and cones. These geometries are the ones demanding most of the measuring effort, for the behavior of mechanisms largely depends on the quality of the surfaces obtained.

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The development of automated measurement systems for monitoring these geometries arises as a response to the increasing automation of manufacture. Nowadays the interest on metrology systems supported by computational geometry is expanding, and is leading to the development of work in different areas.

A major advantage of metrology automation, along with the absence of probe contact and operator intervention, is the high speed of measurements, which provides the acquisition of a large volume of data in a short period of time (Costa *et al.*, 2013). International standards, including ISO 1101, characterize the main functional surfaces, its form deviation and location tolerance. Additionally, various contributions have been proposed, and some may be found in the references (Prisco and Polini, 2010; Samuel and Shunmugam, 2001, 2002, 2003; Fana and Lee, 1999). Computational geometry deals with the systematic study of algorithms and data structures for solving computational geometric problems (Cs.uu.nl, 2015). It appeared by 1970 (Szeliski, 2008), but only after 1995 precipitated the research interest in computer vision systems (Luhmann and Wendt, 2000), due to the cost reduction evolution of computational systems, and to the development of high-resolution digital cameras that was made possible.

The industrial feasibility of metrology based on such systems for geometric shape recognition, depends on the satisfaction of demanding performance criteria, keeping relative cost competitiveness (Mackrory and Daniels, 1995). Their introduction may allow innovative and specific solutions, oriented to industrial automation, with special emphasis on the optimization of the manufacturing process and reduction of what is considered one of the greatest individual costs of production: the inspection process. In parallel, the errors associated with operator intervention, can be also reduced (Chin & Harlow, 1982). Realizing this, the interest from various fields of industrial activities

sparked, from electronic to mechanical components manufacturing, among others, with the sectors mainly related to quality control benefiting from its introduction (Malamas *et al.*, 2003). More recently such systems have also attracted the interest of biomedicine. However, most systems simply deal with large amount of data, without actually performing any recognition of geometric shapes, which is mandatory in many technological areas, where any advances are not possible without a proper corresponding algorithmic support.

The mathematical tools proposed used in the development of the algorithms essentially use differential geometry, taking advantage of the principal curvatures, the mean curvature and the Gaussian curvature. An example is proposed by Ray and Majumder (Ray and Majumder, 1991), for the identification of local invariant features of 3D objects partially occluded. For recognition and localization of 2D shapes, R. Ibrayev Yan and Jia-Bin (Ibrayev and Jia, 2004) introduced a method based on differential and semi-differential invariants, considering data obtained by contact measurement.

Commercially, there is no automated measurement system that can be referred to as a solution for all industrial applications. Nevertheless, there are several systems and proposed algorithms to find the optimal solution of specific cases (Chin and Harlow, 1982; Shakarji, 1998; Benko *et al.*, 2001; Dhanish and Mathew, 2006; Lee, 2009). However, in this domain, a common and desirable property is the recognition in real time in order to allow, for example, control of parts in the manufacturing process. This is the main reason why the algorithms must be fast and robust. Simultaneously, the measurement characteristics of these systems must be checked, in order to minimize the uncertainty of measurement results. The evaluation of these qualities should be made based on reference surfaces.

In current systems, the decision on the

geometric shape is taken by the operator. Based on the coordinates of points, the operator decides whether these belong to one or another surface. The algorithm hereby presented was first proposed by the authors in a previous publication (Costa *et al.*, 2013). The paper begins by presenting the notation and mathematical definition of the problem, showing the decision conditions to recognize the shape and position of the geometries studied. Next, the flowchart of the algorithm is presented, which describes the logical sequence of necessary steps to solve our problem. Subsequently, an application of the algorithm on real parts is shown and the analysis of the results obtained is done. Comparison time spent in measuring is presented to show the advantage of the method. Finally, the conclusions are presented.

2. Notation and mathematical definition of the problem

The algorithm presented in this work uses mathematical tools in the process of recognition and classification of geometric shapes, to allow automatic processing of the data acquired. It was designed to detect itself the geometric shape match and to inform the main data defining its position. It starts by reading a set of discrete data, which can be obtained either by contact or non-contact measurement. However, since the main goal of the application is to automate the measurement process, fast data acquisition of a large set of points must be considered in future, such as image acquisition, for example. Since there is no previous

knowledge of the function that features the surface represented by the acquired data points, a numerical method must be used to obtain the local approximation of the partial derivatives of first and second order at every point. The method chosen was the divided differences. The recognition of the shape type derives from the partial derivatives so obtained, and, in the case of the plane geometry, identification results almost immediately from the first order partial derivatives. Regarding the recognition of the rest of the forms mentioned above, the Gaussian and mean curvatures of the surface were evaluated, using the numerical approximation of the local partial derivatives. The algorithm developed was first tested on data generated in MATLAB, based on the analytical equations of the surfaces under study. It is adaptable to any set of points in a three-dimensional coordinate ordered arrangement. The data corresponding to the three-dimensional coordinates of the points take the format $(x_i, y_j, f(x_i, y_j))$, where $i = 1, 2, \dots, n$, where $j = 1, 2, \dots, p$, and where $z_{(i,j)} = f(x_i, y_j)$ (Costa *et al.*, 2013).

Since the initial data consist in a set of discrete points, the method of divided differences (Valença, 1988) was used in order to determine a numerical approximation to the first and second order partial derivatives at each point.

The Gaussian and mean curvatures at a given point belonging to the surface were calculated, respectively, by Equations (1) and (2).

$$K(i, j) = \frac{\frac{\partial^2 z}{\partial x^2}(i, j) \frac{\partial^2 z}{\partial y^2}(i, j) - \left(\frac{\partial^2 z}{\partial x \partial y}(i, j) \right)^2}{\left(1 + \left(\frac{\partial z}{\partial x}(i, j) \right)^2 + \left(\frac{\partial z}{\partial y}(i, j) \right)^2 \right)^2} \quad (1)$$

$$H(i, j) = \frac{\left(1 + \left(\frac{\partial z}{\partial y}(i, j)\right)^2\right) \frac{\partial^2 z}{\partial x^2}(i, j) - 2 \frac{\partial z}{\partial x}(i, j) \frac{\partial z}{\partial y}(i, j) \frac{\partial^2 z}{\partial x \partial y}(i, j) + \left(1 + \left(\frac{\partial z}{\partial x}(i, j)\right)^2\right) \frac{\partial^2 z}{\partial y^2}(i, j)}{2 \left(1 + \left(\frac{\partial z}{\partial x}(i, j)\right)^2 + \left(\frac{\partial z}{\partial y}(i, j)\right)^2\right)^{3/2}} \quad (2)$$

The recognition of the different geometric shapes was made based upon the satisfaction of the decision conditions presented in Table

1, where r is the radius of the considered shape.

Table 1. Decision conditions for different geometric shapes

	$\frac{\partial z}{\partial x}(i, j) / \frac{\partial z}{\partial y}(i, j)$	$K(i, j)$	$H(i, j)$
Plane	Constant1/Constant2		
Sphere		$1/r^2$	
Cylinder		0	$1/2r$
Cone		0	$1/2r_i$

3. Establishment of the algorithm

A major advantage of the application of computer vision systems to geometrical metrology, along with the absence of contact, is the high speed of measurement,

which provides the acquisition of a large volume of data in a short period of time. The manner these data are acquired represents an important factor in the accuracy of the results.

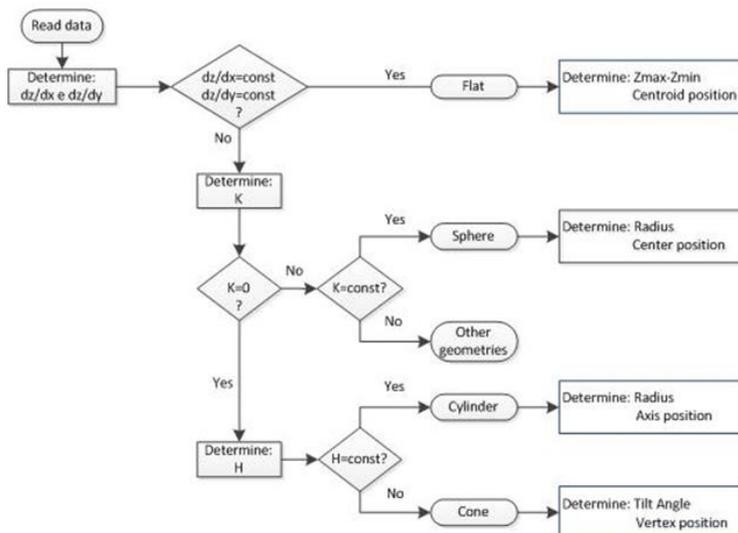


Figure 1. Flowchart of the algorithm

The analysis of equation 1 and 2 (Costa *et al.*, 2013) shows that a deviation from the preset increment h , corresponding to the distance between two consecutive grid points, causes an exponential increase or decrease in the derivative calculation. The treatment of such amount of data demands a suitable algorithm that will be of great importance in the recognition of the functional geometric shapes obtained by mechanical manufacturing. Figure 1 presents the flow chart of the algorithm defining the sequence of steps needed to solve the problem.

3.1. Flat surfaces recognition

Almost all mechanical components have nominally flat surfaces. These surfaces are always characterized by deviations from the theoretical geometric plane, or mathematical plane. There are several factors that contribute to these deviations. The main deviations result from imperfections related with the positioning and manufacturing processes. Uncertainties associated to these factors have been addressed by Minh Hien Bui (Bui, 2011).

The flat surfaces obtained by mechanical manufacturing, present macro geometric irregularities which are generally considered form deviations. According to ISO 1101, the degree of approximation or separation of a real surface, in relation to a nominally flat surface, determines the degree of flatness of that surface.

The recognition algorithm for the planar form, as shown in the flowchart of Figure 2, follows the sequence:

- 1) Point cloud data reading that, structured in a $N \times 3$ matrix;
- 2) First order partial derivatives determination at each point; if these derivatives turn to be constant, then the cloud of data points relate to a flat surface.

3.2. Spherical surfaces recognition

The sphere is also a functional basic geometry in mechanical manufacturing. In industry, the deviation from the spherical shape, or sphericity, has an important effect on the circular motion of components in various machines. Recognition of the spherical shape and the control of its deviation became of paramount importance in mechanical manufacturing. Since international standards, including ISO 1101, do not characterize this deviation explicitly, various contributions have been proposed, and some may be found in the references (Samuel and Shunmugam, 2001, 2002, 2003; Fana and Lee, 1999; Wen and Song, 2004). The recognition of spherical shape and position proposed in this paper is made using the Gaussian curvature of the cloud of points acquired on an actual surface. Following the flow chart in Figure 1, if the first order partial derivatives are not constant, the determination of second order partial derivatives and the Gaussian curvature is performed. When the Gaussian curvature returns a constant value different from zero, the data refers to a spherical surface.

3.3. Cylindrical and conical surfaces recognition

Surfaces of revolution are very common in mechanical construction, either as shafts or as holes. There are several factors contributing to the surfaces, generated by mechanical manufacturing, be not perfect. Storozh *et al.*, presents a work that relates the deviations of cylindrical surfaces with factors involved in the machining process. These authors used a statistical approach to parameterize the uncertainty associated with this relationship (Storozh *et al.*, 2002). Thus, it is often necessary to evaluate the deviation between the actual surface and the mathematically perfect one. ISO 1101 defines the cylindrical shape deviation, or cylindricity, as the tolerance zone between two coaxial cylinders, inside which shall be

contained the real surface. The same rule sets in a similar manner to the conical deviation, or taper, as the tolerance zone between two coaxial cones. This means that all data in the cloud of points should be contained within these tolerance zones.

Thus, continuing to follow the flowchart of Figure 1, if the Gaussian curvature is zero, the calculation of the mean curvature follows. If this one is constant, then the cloud of data points refers to a cylindrical surface. If it is variable, the cloud of data points relates to a conical surface.

4. Application of the algorithm

Nowadays, the global economy is driven by rapid innovation and short lifecycle cycles, with consumer's expectative always increasing in terms of performance, quality and products' cost (VDI 2206). The management of expectations is the driving force for the strong industrial development, especially the technological development in production processes and manufacturing. In this context, computer support and specific algorithms act as aggregators, contributing to increased automation and overall efficiency of manufacturing systems. In this scenario, metrology must be expeditious in its ultimate purpose of, through the global verification of specifications, determine whether the finished product is or not conform. Following these trends, the proposed algorithm aims to give a contribution to the automatic recognition of different functional basic geometries obtained by mechanical manufacturing.

The performance of the algorithm was tested in the recognition and characterization of these surfaces. As discussed above, it was first tested with analytical data in order to obtain confidence in the process. Subsequently, the potential was tested with real data acquired rises surfaces. In this case, were used two different methods to acquire the three dimensional coordinates of points of different basic functional surfaces: CMM

and 3D Scan. The 3D scan, while allowing the rapid acquisition of a large volume of data does not yet characterizes the surface. For example, in the case of a spherical surface, it is not possible with the 3D scan, to obtain the spherical shape, the radius and center location. However, these attributes are in addition to the data, which can be directly obtained by the CMM. Thus, when data are acquired with both devices on the same piece, the proposed algorithm can be applied to the 3D Scan data to characterize the surface and make a comparison with the results obtained in the CMM. Thus, it is possible to study the difference between the results obtained by both methods and, in parallel, the conformance of the requirements for an ideal algorithm (Hoschek and Lasser, 1993):

- Robustness - should determine all geometric characteristics regardless of the type and position of surfaces;
- Numerical Accuracy - must be suitable for the intended application;
- Processing speed - must be compatible with the application, to enable timely responses;
- Auto control - should not require any interactivity, or help the operator for its correct execution

The algorithm was first tested on analytical data generated in MATLAB, in order to get confidence in the procedure. Afterwards it was applied to acquired data of actual simple shapes. The presented model is based on partial derivatives, used to determine the Gaussian and mean curvatures, this values allowed the identification of the geometrical shape regardless of the position it occupied in space.

4.1. Shape and position recognition of flat surfaces

If the determination of the first order partial derivative returns only constant values the identification of a flat surface is immediate.

This determination was made based on a subroutine that satisfies the equations (1).

A simple sub-routine was created to convert and order these data obtained from different devices and formats into three-dimensional coordinates. Figure 2 depicts an example of application using a flat surface of a part shown in Figure 2a, in which the area subjected to measurement using a CMM and Scan 3D was marked. Figures 2b and 2c show the cloud of points associated to those measurements, as the output of the Scan 3D and CMM, respectively. The three-dimensional surface was acquired by the model out of these data points.

Intentionally, the points belonging to line 15, are located in a "V" slot in the measurement

area and are below the plane taken as reference, which, in this case, is coincident with the measured surface. The tolerance specified, determining the total variability to the surface, is then also a deciding factor of the geometric shape. In this particular case, the surface is considered flat when the specified tolerance limits are greater than the distance between points at levels Z_{max} and Z_{min} , in a direction perpendicular to the reference plane. As shown in the flowchart of Figure 1, the decision condition on the flat surface establishes that the first order partial derivative must be constant. Figure 2d and 2e show that these derivatives are constant, both along the axis X and Y, except for the groove where dz/dx expectedly changes.

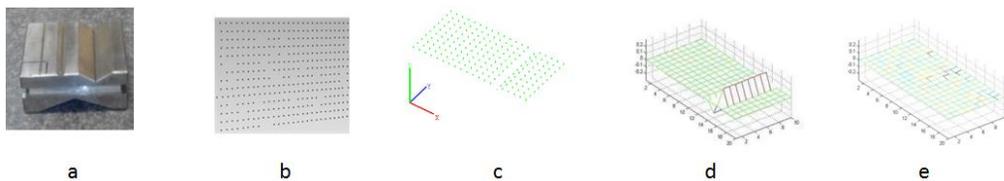


Figure 2. Planar surface handling. a) Actual surface area subjected to measurement (bordered in black). b) Cloud points obtained with a 3D scan; c) Cloud points obtained with a CMM. Plane decision conditions: d) - $dz/dx = \text{constant}$; e) $dz/dy = \text{constant}$

The spatial position of the flat shape is sufficiently defined by a plane parallel to the data set, containing the centroid of the eligible data set, the plane versor and by the directions relative to the axes X, Y and Z.

4.2. Shape and position recognition of spherical surfaces

In industry, the deviation from the spherical shape, or sphericity, has an important effect on the circular motion of components in various machines. Therefore, defects such as roughness, curling or shape can result in the generation of a large amount of heat, causing a rise in the surface temperature of the components involved, resulting in wear and life reduction. Thus, recognition of the spherical shape and the control of its

deviation becomes of paramount importance in mechanical manufacturing.

The analysis of the algorithm shows that, when constant values for the first order partial derivatives are not exclusively returned, the decision on the flat surface is denied, the determination of the second order partial derivatives starts, and then the Gaussian curvature (K) at each point is evaluated using Equation (1).

Figure 3a shows a CMM standard ball, in which the acquisition of the point cloud shown in Figures 3b and 3c were performed using a Scan 3D and CMM, respectively. When the partial derivatives of the first order are not constant, the Gaussian curvature must be constant and different from zero (Fig. 3d).

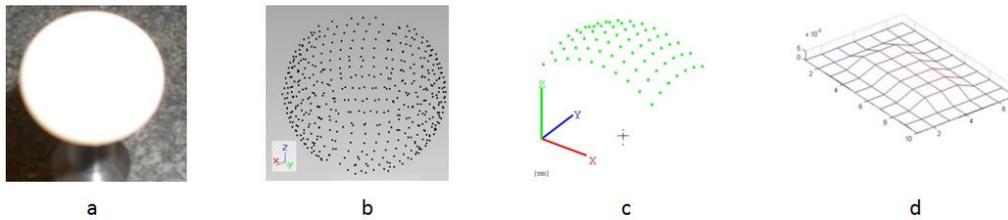


Figure 3. A spherical surface subject measurement. a) Spherical surface measured (CMM standard ball). b) Cloud points obtained with a 3D scan; c) Cloud points obtained with a CMM; d) Spherical shape decision condition: $K = \text{nonzero constant}$

The attributes of a spherical shape may be sufficiently defined by the value of its radius, calculated by the Gaussian curvature, referred as k (Eq. 3), and the coordinates of its center, referred as C . So the position

$$C_{i,j} = P_{i,j} - \frac{1}{\sqrt{k_{i,j} \|u\|}} u_{i,j} \quad (3)$$

$$u_{i,j} = \left(\frac{\partial z}{\partial x} \Big|_{P_{i,j}}, \frac{\partial z}{\partial y} \Big|_{P_{i,j}}, -1 \right) \quad (4)$$

problem can be solved by determining the average center position (Eq. 3), which was calculated based on the normal vector (Eq. 4) at each data point.

4.3. Shape and position recognition of cylindrical and conical surfaces

The interest in the calculation of K lies in the fact that it expresses an invariant feature of the surface at each point. Then, if K has a zero value, the next step is the calculation of the mean curvature (H) using equation (4). If

H is constant, then the surface is cylindrical; otherwise, the surface is conical. Figure 4a shows the cylindrical part in which the data acquisition was performed, and the clouds of points shown in Figures 4b and 4c relate, respectively, to the acquisition by 3D Scan and CMM.

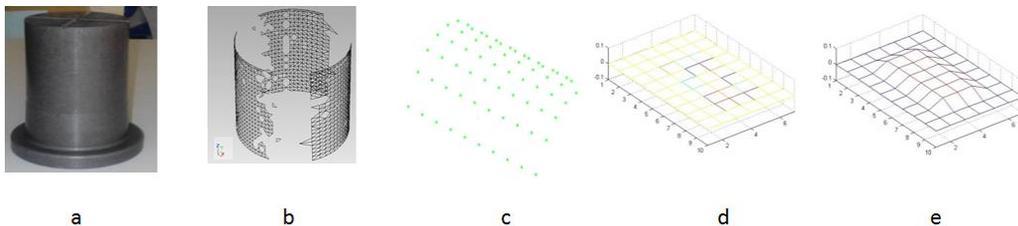


Figure 4. Cylindrical surface measurement. a) Cylindrical surface measured. b) Mesh obtained with a 3D scan; c) Cloud points obtained with a CMM; Cylindrical shape decision condition: d) $K = 0$; e) $H = \text{nonzero constant}$

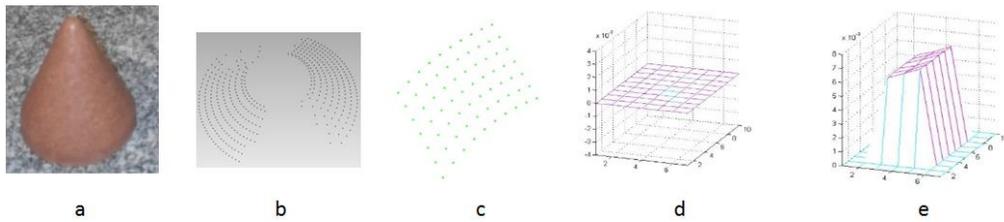


Figure 5. Conical surface measurement. a) Conical surface measured. b) Mesh obtained with a 3D scan; c) Cloud points obtained with a CMM; Conical shape decision conditions: d) $K = 0$; e) $H = \text{variable}$

The Gaussian curvature being null (Fig. 4d), the mean curvature is constant and different from zero (Fig. 4e). The radius was determined based on the mean curvature using the equation $H = 1/2r$. The position of the cylindrical shape is sufficiently defined by an axis point and the angles that this forms with the coordinate axes. The axis point was calculated by adding to the coordinates of a point P, belonging to the surface, the normalized versor multiplied by the average radius of the cylinder.

Figure 5 shows the case of a conical part (Fig. 5a), where the point clouds acquired are shown in Figures 9b and 9c. Figures 9b.1 and 9c.1 show the three-dimensional surface acquired by the model out of these data points.

The decision condition on the conical surfaces states that, the Gaussian curvature being null (Fig. 5a), the mean curvature is variable (Fig. 5b).

The position of the conical shape is sufficiently defined by the coordinates of the vertex and the angles that its axis forms relatively to the coordinate axes.

5. Analysis of results

The cloud data points generated in MMC, resulted from actual touching on real surfaces with the geometric shapes desired in this study. With the knowledge acquired by the analytical results the model was then tested over the acquired data by CMM. The CMM point cloud data have resulted of palpation, by contact, on the real surface. In

this acquisition, the radius (r) of the used ruby ball was 1.0 mm and a sphericity of 0,08 μm or less, which was used to compensate the obtained results. Fig. 6, shows an example of this compensation.

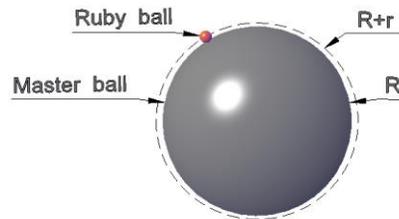


Figure 6. Radius compensation of ruby ball

Initially, these geometric attributes were obtained by the MMC software and subsequently by the model developed. At this stage, for the sake of simplicity, the results obtained with the MMC are conventionally considered correct. The recognition of different forms was tested by generating the respective surfaces (Figs. 2c, 3c, 4c and 5c) and by the decision conditions checking (Figs. 2d, 2e, 3d, 4d, 4e, 5d, 5e). The model demonstrated good robustness in the recognition of all geometric shapes treated.

The results obtained are presented in Table 2, and validate the suitability and potential of the algorithm proposed for the identification of the shape and spatial position of the geometric surfaces studied. The results obtained are presented in Table 2, and validate the suitability and potential of the algorithm proposed for the identification of

the shape and spatial position of the geometric surfaces studied.

Since inspection is generally a time consuming task, besides reducing the operator intervention in the process, the ultimate objective of this method is to reduce the overall measuring time. Taking as an example the conical shape measured, in Table 3 the time spent in the measurement using the conventional CMM machine and the 3D Scan machine. Using 3D Scan machine, the acquisition ratio is about 20 times faster than using the conventional

CMM machine. For the identification of the shape and position of a part the accuracy obtained by the first method (3D Scan) is enough, but, if it is not the case, accurate measurements can be obtained after a fast shape and position recognition. Having this statement consideration, and also taking in account that nowadays digital photography is available at very low cost, the next step will be to use it to build a cloud point Cartesian coordinates mapping of the shape, and then perform the shape and position identification.

Table 2. Comparison of results between the two measurement systems

		CMM	3D Scan
Plane	Flatness [mm]	0.4799	0.4760
	Centroid [mm]	unavailable	14.4998; 11.4991; -0.0219
Sphere	Sphericity	0.0158	0.0000
	Radius [mm]	15.9834	15.6848
	Center [mm]	0.0035; -0.0003; 120.8925	0.0000; 0.0000; 121.2699
Cylinder	Cylindricity	0.0432	0.0782
	Radius [mm]	30.5005	29.5118
	Axis angles [°:':"]	27°27'14"; 69°33'40" 72°28'52"	27°19'24"; 69°43'56"; 72°28'20"
	Axis (Point) [mm]	101.9976; 38.7528; 35.2881	10.0899; 56.6629; 4.5321
Cone	Conicity	0.0804	0.1004
	Tilt Angle [°:':"]	47°:53':00"	48° 4' 36"
	Axis angles [°:':"]	177°50'08"; 89°06'11" 91°58'11"	89°55'31"; 89°53'11" 179°46'21"
	Axis (Vertex) [mm]	unavailable	11.1631; 1.9996; 8.6904

It is expected that the overall time spent be even shorter. Industrial measuring equipment should then be equipped with 3D vision systems to determine shape and position

determination, and then accurate measurements can be performed, if and were necessary.

Table 3. Comparison of the time spent in the measurement by both methods

	Preparation time [min]	Acquisition time [min]	Acquired points	Acquisition ratio [Points/min]
CMM	10	5	70	4.7
Scan 3D	0.25	3	304	93.5

6. Conclusions

The algorithm proposed in this paper was developed intending to be flexible and adaptable to different data acquisition systems. The data simply must be converted to three-dimensional coordinates and structured in the form of an $N \times 3$ matrix. The use of Gaussian and mean curvatures proved very effective in the decision-making algorithm. These values are intrinsic to each geometry and are invariant to the position it occupies in space. The verification of the decision conditions and the results validated the suitability and potential of the proposed algorithm. The advantage of the proposed algorithm in the treatment of acquired data is to be able to recognize and to classify the shape and position of basic functional geometries reducing, or even without, the use of the operator, i.e., in an automatic mode. The resource to the versors at each point of the studied surfaces was effective in

determining the position attributes for any position in tridimensional space.

The obtained results allowed to conclude that the robustness of the model depends on the quality of the data acquisition. The comparison of results between the measurement systems show that the model behaves in a similar mode either with CMM data or with 3D scan data. Comparison time spent in measuring show the advantage of the method, opening further developments tacking advantage of fast acquisition methods.

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References:

- Benkő, P., Martin, R., & Várady, T. (2001). Algorithms for reverse engineering boundary representation models. *Computer-Aided Design*, 33(11), 839-851. doi:10.1016/s0010-4485(01)00100-2
- Bui, M.H. (2011). Identification et simulation des incertitudes de fabrication. Retrieved 2 February 2015, from http://tel.archives-ouvertes.fr/docs/00/69/72/04/PDF/35609_BUI_2011_archivage.pdf
- Chin, R., & Harlow, C. (1982). Automated Visual Inspection: A Survey. *IEEE Transactions On Pattern Analysis And Machine Intelligence*, PAMI-4(6), 557-573. doi:10.1109/tpami.1982.4767309
- Costa, C.A., Silva, J.M., Monteiro, A., & Filipe, A.I. (2013). *A contribution to recognize shape and position of basic functional geometries obtained by mechanical manufacturing*. 22nd International Congress of Mechanical Engineering, COBEM 2013, November 2013, Ribeirão Preto, SP, Brazil
- Cs.uu.nl. (2015). *Computational Geometry, Algorithms and Applications.*, Retrived from <http://www.cs.uu.nl/geobook/>, Accessed 2 February 2015.
- Dhanish, P., & Mathew, J. (2006). A fast and simple algorithm for evaluation of minimum zone straightness error from coordinate data. *Int J Adv Manuf Technol*, 32(1-2), 92-98. doi:10.1007/s00170-005-0317-1
- Fana, K., & Lee, J. (1999). Analysis of minimum zone sphericity error using minimum potential energy theory. *Precision Engineering*, 23(2), 65-72. doi:10.1016/s0141-6359(98)00024-5

- Hoschek, J., & Lasser, D. (1993). *Fundamentals of computer-aided geometric design*. Wellesley, Mass.: A.K. Peters.
- Ibrayev, R., & Jia, Y.-B. (2004). *Tactile Recognition of Algebraic Shapes Using Differential Invariants*. Proceedings of the 2004 IEEE International Conference on Robotics & Automation, New Orleans
- Lee, M. (2009). An enhanced convex-hull edge method for flatness tolerance evaluation. *Computer-Aided Design*, 41(12), 930-941. doi:10.1016/j.cad.2009.06.011
- Luhmann, T., & Wendt, K. (2000). Recommendations for Acceptance and Verification Test of Optical 3-D Measurement Systems. Retrieved from: http://www.isprs.org/proceedings/xxxiii/congress/part5/493_XXXIII-part5.pdf, Accessed 10 Dec. 2012.
- Mackrory, J., & Daniels M. (1995). The Impact of New Technology in Machine Vision". 17 Retrieved from: <http://dx.doi.org/10.1108/EUM0000000004257>, Accessed Dec. 2012
- Malamas, E., Petrakis, E., Zervakis, M., Petit, L., & Legat, J. (2003). A survey on industrial vision systems, applications and tools. *Image And Vision Computing*, 21(2), 171-188. doi:10.1016/s0262-8856(02)00152-x
- Prisco, U., & Polini, W. (2010). Flatness, Cylindricity and Sphericity Assessment Based on the Seven Classes of Symmetry of the Surfaces. *Advances In Mechanical Engineering*, 2(0), 154287-154287. doi:10.1155/2010/154287
- Ray, K., & Majumder, D. (1991). Recognition and positioning of partially occluded 3-D objects. *Pattern Recognition Letters*, 12(2), 93-108. doi:10.1016/0167-8655(91)90055-q
- Samuel, G., & Shunmugam, M. (2001). Evaluation of sphericity error from coordinate measurement data using computational geometric techniques. *Computer Methods In Applied Mechanics And Engineering*, 190(51-52), 6765-6781. doi:10.1016/s0045-7825(01)00220-1
- Samuel, G., & Shunmugam, M. (2002). Evaluation of sphericity error from form data using computational geometric techniques. *International Journal Of Machine Tools And Manufacture*, 42(3), 405-416. doi:10.1016/s0890-6955(01)00124-9
- Samuel, G., & Shunmugam, M. (2003). Evaluation of circularity and sphericity from coordinate measurement data. *Journal Of Materials Processing Technology*, 139(1-3), 90-95. doi:10.1016/s0924-0136(03)00187-0
- Shakarji, C. (1998). Least-squares fitting algorithms of the NIST algorithm testing system. *Journal Of Research Of The National Institute Of Standards And Technology*, 103(6), 633. doi:10.6028/jres.103.043
- Storozh B.D., Karpyk R.T., & Storozh, Y.B. (2002). *The relations of accuracy parameters of cylindrical surfaces*. The international meeting of the Carpathian region specialists in the field of gears. Serie C. Vol. X11 - Baia Mare, North University, p.153-160
- Szeliski, R. (2008). Computer Vision: Algorithms and Applications. Retrieved from: http://research.microsoft.com/en-us/um/people/szeliski/book/drafts/szeliski_20080330am_draft.pdf, Accessed 19 Dec. 2012
- Valença, M.R. (1988). *Métodos Numéricos*. Instituto Nacional de Investigação Científica, Braga, Portugal
- Wen, X., & Song, A. (2004). An immune evolutionary algorithm for sphericity error evaluation. *International Journal Of Machine Tools And Manufacture*, 44(10), 1077-1084. doi:10.1016/j.ijmactools.2004.02.018

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