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FOSTERING CREATIVITY IN STUDENTS VIA MATHEMATICAL LEARNING

Abstract: *Developing students' creative abilities is becoming an increasingly relevant task in the context of a rapidly changing world and growing demands for professional skills. As a fundamental science, mathematics provides unique opportunities for forming creative thinking and analytical abilities in schoolchildren. This study aims to examine methods for developing students' creative abilities through the study of mathematics, with a particular emphasis on the use of geometric problems. The research methodology included theoretical analysis of scientific literature, comparative analysis of various techniques, conducting a pedagogical experiment, questionnaires and interviews with students and teachers, and systematic observation of the educational process. As a result of the study, it was found that the use of geometric problems and the integration of problem-based learning, project activities, group work, and information and communication technologies significantly contribute to the development of students' creative thinking and analytical abilities. The experimental group of students showed significant improvement in results compared to the control group, confirming the effectiveness of the proposed methods. The practical significance of the results lies in the possibility of applying the developed methods in educational practice to improve the quality of mathematics education and student motivation. Implementing these methods creates a stimulating and supportive learning environment that fosters the development of student's creative potential and critical thinking, which are essential for their successful academic and professional activities.*

Keywords: *Creative abilities, Mathematical education, Creative thinking, Geometric problems, Problem-based learning, Project activities, Information and communication technologies*

1. Introduction

The modern education system specialises in developing students' creative abilities, as these skills facilitate successful adaptation in a rapidly changing world. Mathematics, one of the oldest and most important sciences, not only fosters logical thinking but also is a powerful tool for developing creative potential. One of the current issues in modern pedagogy is integrating methods that develop students' creative abilities in learning mathematics. The relevance of this study is driven by the need to find new approaches to teaching that promote the development of creative thinking, the capacity for innovation, and a non-standard approach to problem-solving. The scientific

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value of this work lies in the systematisation and analysis of methodologies that facilitate the development of creative abilities through mathematical education. The theoretical significance of the study lies in expanding the understanding of the mechanisms for developing creative thinking in the process of learning mathematics, while the practical significance lies in the possibility of applying the obtained results in pedagogical practice. The foundation of our research is the assumption that the use of geometric problems and the integration of analytical and creative approaches to their solution promote the development of spatial thinking, imagination, and critical skills in students. Such tasks allow students to view mathematical problems from different perspectives, find original solutions, and develop the ability to think abstractly. This article aims to identify and characterise effective methods for developing students' creative abilities through the study of mathematics. An analysis of scientific works shows that many scholars have researched the issue of developing creative abilities through mathematics teaching. However, most have focused on general approaches, leaving out specific aspects of using geometric problems. In this work, we aim to fill these gaps by examining in detail how geometry can be used to stimulate creative thinking.

In modern society, where technology is rapidly advancing, and the demands for professional skills are constantly changing, the development of student's creative abilities is becoming particularly significant. Creative thinking is critical to success across various fields, including science, technology, art, and business. In this context, mathematics, as a fundamental science, provides unique opportunities to develop schoolchildren's creative potential. However, despite recognising the importance of creative thinking, traditional mathematics teaching methods often focus on the mechanical assimilation of formulas and algorithms, leading to insufficient student motivation and limiting their creative potential. Teachers often face the problem of a lack of interest in the subject and the inability of students to apply mathematical knowledge in non-standard situations.

A pressing issue in modern pedagogy is the search for and use of effective methods that ensure the mastery of mathematical knowledge and promote the development of student's creative abilities. The use of geometric problems, which require students to employ logical, spatial, abstract thinking and creativity, deserves special attention. To date, the integration of creative methods into mathematics teaching is insufficiently covered in the scientific and methodological literature. There is a need to develop new pedagogical strategies that effectively stimulate the development of creative thinking in students, increase their interest in studying mathematics, and motivate them to participate actively in the learning process. Therefore, the problem addressed in this article lies in the insufficient development and implementation of mathematics teaching methods to develop students' creative abilities. The need to systematise existing approaches and develop new methodologies focused on using geometric problems determines this research's relevance.

This study aims to investigate and demonstrate methods of developing students' creative abilities by learning mathematics, focusing on geometry problems. The article discusses how using analytical and creative approaches to solving mathematical problems contributes to developing students' imagination, spatial thinking and critical skills.

Analysis of methodological principles: To consider the theoretical aspects of developing creative abilities through mathematics education based on modern scientific research and pedagogical practices.

1. To provide specific examples of geometric problems that promote the development of creative thinking and analytical skills in students.

2. To propose methods and strategies for teaching mathematics to stimulate students' creative activity.

3. To conduct an experimental study to determine the effectiveness of the proposed methods in developing students' creative abilities.

4. To evaluate the results of the experimental study, analyse the data obtained and draw conclusions about the impact of learning mathematics on the development of creative abilities.

5. To develop recommendations for teachers on introducing effective methods of teaching mathematics, invested in expanding students' creative abilities.

These goals and objectives aim to improve the quality of mathematics education and develop the skills necessary for students to solve non-standard problems successfully and apply a creative approach in various fields of activity.

2. Literature review

2.1 Theoretical aspects of developing creative abilities

The research by Bevz (2005), Kushnir (2007; 2017), Peteczuk (1992), and Sepulcre and Vidal (2019) thoroughly examines the fundamental theoretical foundations for developing students' creative abilities through mathematics education. They emphasise the importance of geometric problems, which contribute to developing logical and spatial thinking. These tasks involving construction and proof of theorems encourage students to engage in analysis and abstraction, essential aspects of creative thinking.

2.2 Problem-based learning and creative thinking

The works of Dash and Panda (2021), Khalid et al. (2020), Rahman et al. (2020), Sari et al. (2018), and Fitrianawati et al. (2020) demonstrate the effectiveness of problem-based learning in developing creative thinking. They note that presenting complex and engaging tasks that require an unconventional approach stimulates students to seek solutions and develop creative skills independently. These studies confirm that problem-based learning significantly enhances students' motivation and interest.

2.3 Group work and interaction

The studies by Sipayung et al. (2021), Muttaqin et al. (2021), Yayuk et al. (2020b), and Adiastuty et al. (2020) highlight the importance of group work and interaction for the development of creative abilities. They emphasise that collaboratively solving tasks promotes the exchange of ideas and the development of communication skills, which are essential for the creative process. Students working in groups learn to view tasks from different perspectives and propose original solutions.

2.4 Project activities and practical application

The works of Telegina et al. (2019), Adiastuty et al. (2020), Madihah et al. (2020), Batsurovska et al. (2021), Saputri et al. (2022) and Dotsenko et al. (2023) highlight the importance of project activities in the educational process. They show that projects requiring the application of mathematical knowledge to solve real-world problems stimulate students' research activities and the development of creative thinking. Project activities allow students to practically apply theoretical knowledge, contributing to a deeper understanding of the subject and developing creative abilities.

2.5 Information and Communication Technologies (ICT)

The studies by Kholil (2020), Syaiful et al. (2020), Fitrianawati et al. (2020), Setyana et al. (2019), Sipahutar et al. (2017), and Haji & Yumiati (2021) examine the use of information and communication technologies in mathematics education. They note that using specialised software for modelling and visualising mathematical processes allows students to experiment and find new approaches to problem-solving. These technologies make the learning process more interactive and engaging, promoting creative thinking development.

2.6 Empirical studies and experimental data

The works of Yayuk et al. (2020a), Musri & Syukri (2020), Haji & Yumiati (2021), Tetty et al. (2021), Li & Qiao (2016), and Khalid et al. (2020) present the results of experimental studies that confirm the effectiveness of the proposed methodologies. They show that students who were taught using creative and interactive methods demonstrated significantly higher results in developing creative abilities than those who were taught using traditional methods. These studies emphasise the importance of a comprehensive approach to teaching, which includes various methods and technologies to stimulate creative thinking.

Therefore, the analysis of scientific sources shows that the development of students' creative abilities through mathematics is possible due to the integration of various methods and strategies, such as problem-based learning, project activities, group work, and ICT. These methods increase students' motivation, interest, and creative potential, as confirmed by theoretical and empirical evidence.

3. Research methods

3.1 Characterisation of the sample

To achieve the aims and objectives of our study, we used a comprehensive approach that includes theoretical and empirical methods. The following methods were applied in the research process:

Theoretical methods:

- Analysis of scientific literature: study and systematisation of modern research and publications on the development of student's creative abilities through the study of mathematics, emphasising the use of geometric tasks. Analysis of pedagogical and psychological theories related to creative thinking and mathematics education.
- Comparative analysis: comparing different methods and approaches to teaching mathematics to identify the most effective strategies for developing students' creative abilities.
- Generalisation and systematisation: summarising and systematising the theoretical data obtained through practical recommendations and methods.

Empirical methods:

- Pedagogical experiment: conducting an experimental study in educational institutions to assess the effectiveness of the proposed methods. The experiment included forming an experimental and a control group, introducing innovative methods into the educational process of the experimental group and traditional teaching methods in the control group.
- Questionnaires and interviews: developing and conducting questionnaires and interviews with students and teachers to obtain qualitative information about the perception and effectiveness of the proposed methods. The questionnaires included questions about motivation, interest in the subject, perception of geometry tasks and levels of creative thinking.

- Observation: systematic observation of the learning process in the experimental and control groups to assess students' activity, engagement and creativity. The observation was conducted using specially developed protocols and checklists.
- Testing: preliminary, intermediate and final testing of students to assess changes in their creativity and logical and spatial thinking. Testing included performing standard and creative geometry tasks.
- Data analysis: processing and analysing the collected data using statistical analysis methods to identify significant differences between the experimental and control groups. Evaluation of the experiment's results and formulation of conclusions based on the data obtained.

Applying a comprehensive approach to the study allowed us to assess the effectiveness of the proposed methods comprehensively and obtain reliable data on the impact of geometric tasks on the development of student's creative abilities.

4. Research results

Developing students' creative abilities through mathematics education is based on several theoretical aspects confirmed by modern scientific research and pedagogical practices.

1. Integration of creative tasks into the learning process. Modern research shows that integrating creative and research tasks into the mathematics learning process stimulates students to think unconventionally and find new solutions. Creative tasks help students consolidate basic knowledge and develop the ability for abstract thinking and idea generation.

2.Problem-based learning method. Problem-based learning, which is based on setting and solving complex and exciting tasks, promotes the development of creative skills. This method involves students independently seeking ways to solve problems, stimulating their creative and critical thinking. Students learn to analyse situations, synthesise information, and produce original solutions when solving tasks.

3. Group work and discussions. Organising group work and conducting discussions on mathematical topics allows students to exchange ideas, jointly solve tasks, and develop communication and creative skills. Joint discussions of tasks lead to the emergence of new ideas and approaches to solving problems, which is an essential aspect of creative thinking.

4. Project activities. Including project activities in the learning process helps students apply mathematical knowledge to solve practical and research tasks. Projects can aim to study real-world problems and solve them using mathematical methods, stimulating students to take a creative approach and develop research skills.

5. Use of Information and Communication Technologies (ICT). Modern ICT provides many opportunities for developing students' creative abilities. Using specialised software for modelling, simulations, and visualising mathematical processes allows students to experiment, find new approaches, and visualise their ideas.

6. Motivation and support for students. An important aspect is creating a motivational environment encouraging students to take a creative approach and engage in research activities. Support and encouragement from the teacher, as well as freedom in choosing methods and ways to solve tasks, contribute to developing students' creative potential.

7. Active learning methods. Active learning methods, such as mathematical games, quests, and competitions, increase students' interest in the subject and stimulate their creative activity. Game-based methods allow

students to immerse themselves in learning and developing creative thinking in an informal setting.

8. Development of critical thinking. Modern pedagogical practices emphasise the importance of developing critical thinking through mathematics education. Analysing and evaluating different approaches to solving tasks and critically reflecting on their own decisions contribute to developing a conscious and creative approach to problems.

These theoretical aspects emphasise the importance of a comprehensive approach to teaching mathematics, including integrating creative tasks, using modern technologies and active learning methods, and creating a motivational environment for developing students' creative abilities.

Introducing geometric problems into the learning process is crucial in developing students' creative thinking and analytical skills. These tasks require students to see and analyse spatial relationships, which fosters the development of their abstract thinking. Solving complex geometric problems helps students form logical thinking skills and a systematic approach to problem-solving (Nahrowi, & Hobri, 2020). Additionally, such tasks often require unconventional approaches, which stimulate creative thinking. Using geometric problems in teaching also develops the ability to visualise and spatial imagination.

The examples of tasks below (the New "Bipedal" Theorem of Equivalence) demonstrate how integrating geometry into the learning process promotes the development of critical cognitive and creative skills in students.

Example 1

A triangle *ABC* is inscribed in a circle with a centre *O*. The point *X* belongs to the side *BC* ($X \in BC$).

The point *X* is connected to the vertex *A* of the triangle *ABC*.

The ray AY is angled $\angle CAX$ to intersect the described circle around the triangle *ABC* ($\angle CAX = \angle BAY$, $Y \in (O; R = OA)$) (Figure 1).

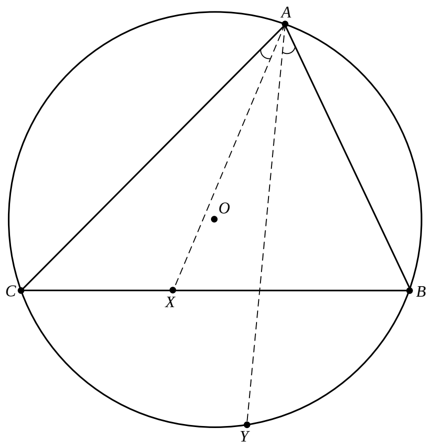

Figure 1

From the point X to the sides AC and AB of the triangle ABC, let's omit the perpendiculars XMX and XNX, respectively (Figure 2).

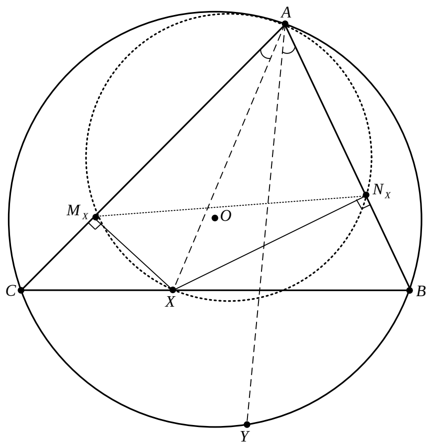

Figure 2

In the following, the segments XM_X and XN_X will be called pedals of the point *X* or bipeds of the point *X*, since we have two perpendiculars.

A circle AN_XXN_X with diameter can be described around the quadrilateral *AX*.

So,

$M_X N_X = AX \sin \angle A = AX \sin \angle BAC.$ (1)

Form a quadrilateral AM_XYN_X (Figure 3) and prove that its area is equal to the area of the triangle *ABC*. Otherwise: prove that $S_{AM_XYN_X} = S_{ABC}$

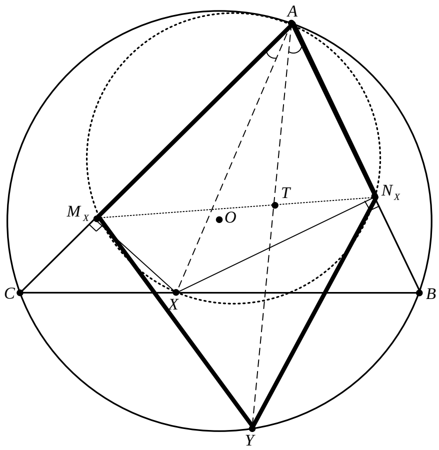

Figure 3

Proof. The area of a quadrilateral AM_XYN_X has the following general form:

$S_{AM_XYN_X} = \frac{1}{2} AY * M_X N_X * sin \angle ATN_X.$(2),

where $AY \cap M_X N_X = T$.

As shown in Figure 2: $\angle AN_X M_X = \angle AXM_X$, because they are based on the same arc AM_X.

$\angle CAX = \angle BAY$ (by construction).

It follows that triangles ATN_X and $AM_X X$ are similar, and therefore, $\angle AM_X X = ATN_X = 90°$.

And then,

$S_{AM_XYN_X} = \frac{1}{2} AY * M_X N_X * \sin 90° = \frac{1}{2} AY * M_X N_X$.

$M_X N_X = AX \, sin \angle BAC.$ (3)

You can write the formula for the area of a quadrilateral AM_XYN_X differently:

$S_{AM_XYN_X} = \frac{1}{2} AY * AX \sin \angle BAC.$ (4)

Triangles *ACX* and *AYB* are similar (Figure 4) ($\angle CAX = \angle BAY$ by construction, $\angle ACB = \angle AYB$ are based on the same arc).

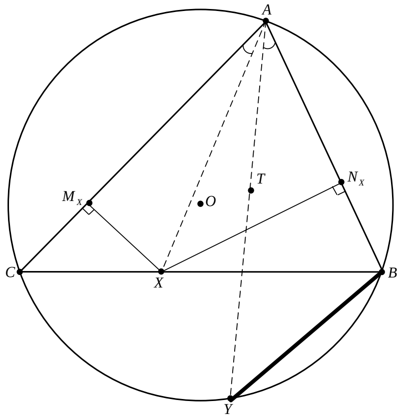

Figure 4

We have:

$\frac{AC}{AY} = \frac{AX}{AB}.$ (5)

$AC * AB = AX * AY.$ (6)

Otherwise: the area of the triangle *ABC*:

$S_{ABC} = \frac{1}{2} AC * AB \sin \angle BAC.$ (7)

Area of a quadrilateral AM_XYN_X:

$S_{AM_XYN_X} = \frac{1}{2} AX * AY * \sin \angle BAC.$

Considering (6), then

$S_{AM_XYN_X} = S_{ABC}.$ (8)

Proven.

We obtained and proved *the first equality* of a quadrilateral and a triangle (1) using the isogonality of the lines *AX* and *AY* (isogonal lines are lines that form equal angles with the lines of a triangle and pass through a common point, the vertex of the triangle).

A problem with a similar situation with isogonals *AX* and *AY* is presented in Appendix 1.

Example 2

Consider the case when the point *X* of the triangle *ABC* coincides with the base of the internal bisector of the angle $\angle BAC$: $X = L_1$: (Figure 5).

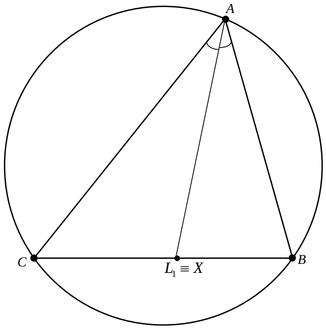

Figure 5

In this case, the isogonal AX to is the continuation of the internal bisector AL_1 until it intersects the described circle around the triangle ABC at the point W_1. Otherwise: $W_1 = Y$ (Figure 6).

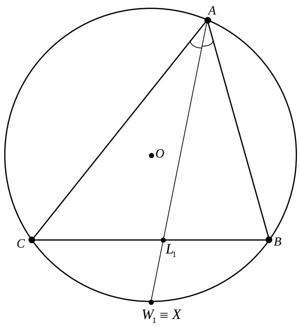

Figure 6

To form a quadrilateral, similar to an equilateral triangle

(1):

$$S_{ABC} = S_{AM_XYN_X} = S_{AM_LW_1N_L}, \; (9)$$

where $M_LL_1 \perp AC$, $N_LL_1 \perp AB$ (Figure 7).

Otherwise, the area of the quadrilateral $AM_LW_1N_L$ is equal to the area of the triangle ABC:

$$S_{ABC} = \frac{1}{2} AW_1 * M_LN_L$$

− the author of this problem is *Isaak Arkadiiovych Kushnir* ("The Triumph of School Geometry").

To form a quadrilateral, similar to an equilateral triangle (2):

$$S_{ABC} = S_{AM_YXN_Y} = S_{AM_WL_1N_W}, \qquad (10)$$

where $W_1M_W \perp AC$, $W_1N_W \perp AB$ (Figure 8).

Otherwise: the quadrilateral $AM_W * L_1N_W$ is equal to the triangle *ABC*.

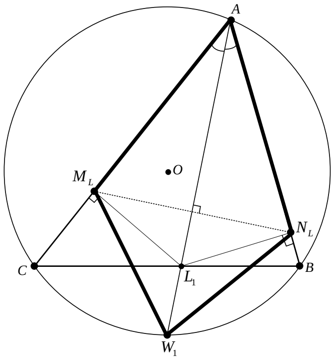

Figure 7

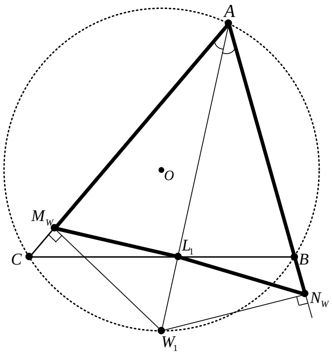

Figure 8

Example 3

Consider the case when the point X of the triangle *ABC* coincides with the base of the height drawn from the vertex A to the side *BC*: $X = H_1$ (Figure 9).

$$\angle BAH_1 = 90° - \angle CBA. \qquad (11)$$

AX − isogonal to AY.

$\angle BAH_1 = \angle CAY = 90° - \angle B$, i.e. AY coincides with the diameter AA_0 of a triangle *ABC* (triangles ABH_1 and AA_0C are similar).

To form a quadrilateral, similar to an equilateral triangle (1):

$$S_{ABC} = S_{AM_YXN_Y} = S_{AM_HA_0N_H}, \; (12)$$

where $H_1M_H \perp AC$, $H_1N_H \perp AB$ (Figure 10).

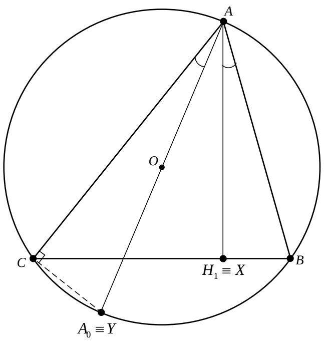

Figure 9

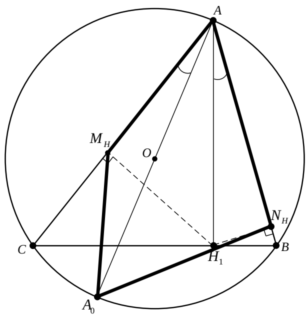

Figure 10

Otherwise, a quadrilateral $AM_HA_0N_H$ is equal to the triangle ABC.

A universal method for finding unexpected proofs of geometric formulas and equivalence of figures has been discovered.

Using geometric problems in teaching develops students' creative thinking and analytical abilities. These tasks contribute to the formation of logical thinking skills and abstract analysis. The application of unconventional approaches stimulates creative thinking and the ability to visualise. As a result, students become better prepared to solve complex problems and develop their cognitive and creative skills.

The methodologies and strategies for teaching mathematics to stimulate students' creative activity are presented in Table 1.

Table 1. Methods and strategies for teaching mathematics to stimulate students' creative activity

Methods	Strategy
Integration of project activities	*Project tasks:* include projects that require students to apply mathematical knowledge to real-world problems. For instance, students might investigate how mathematics is used in architecture, engineering, or the environment. *Collaborative projects* involve organising group projects where students work together to solve complex problems. These projects develop collaboration skills and stimulate collective creativity.
Method of problem-based learning	*Problem-based learning:* setting tasks for students that require a non-standard approach and creative thinking. For instance, presenting problems that do not have an obvious solution and encouraging students to find different ways to solve them. *Debates and discussions:* organising debates on mathematical problems where students can present and discuss their ideas with their classmates.
Use of Information and Communication Technologies (ICT)	*Interactive apps:* using software for modelling and visualising mathematical processes, such as GeoGebra or Desmos, to allow students to experiment and visualise their ideas. *Online courses and resources:* providing access to online courses and resources that offer interactive tasks and projects.
Active learning and game methods	*Mathematical games:* incorporating mathematical games and quests into the learning process to make learning fun and exciting. Games can be aimed at solving logical problems and developing creative thinking. *Competitions and olympiads:* organising mathematical competitions and olympiads that encourage students to be creative and work independently on complex tasks.

Group work and cooperation	*Small group work:* dividing students into small groups to solve problems, which promotes the exchange of ideas and the development of creativity. *Masterclasses and seminars:* holding masterclasses and seminars with the participation of mathematics experts who can show non-standard approaches to solving problems.
Individual research projects	*Research tasks:* encouraging students to conduct their own research on mathematical topics of interest. This may include collecting data, analysing and presenting results. *Scientific papers:* encouraging the writing of scientific papers and articles in which students can present their research and discoveries.
Methods of visualisation and spatial thinking	*Graphical tasks* require creating graphs, charts, and other visual representations to help students better understand and analyse mathematical concepts. *Modelling and drawing:* using modelling and drawing to solve geometry problems, which develops spatial imagination and creative thinking.
Feedback and support	*Individual consultations:* providing regular feedback and individual consultations to support and encourage students in their creative efforts. *Portfolio of achievements:* introducing a portfolio where students can record their achievements and progress in solving creative tasks.

Source: Compiled by the author

These methods and strategies aim to create a stimulating and supportive environment that promotes students' creativity and interest in mathematics.

The present study describes an experimental investigation designed to assess the efficacy of the proposed methods.

Research objective: To determine the effectiveness of the proposed methods and strategies for teaching mathematics in developing students' creative abilities. The study involved students from grades 7-11 from two schools, with 89 participants. The students were divided into experimental and control groups.

Research methodology: Initial testing to assess the baseline level of students' creative abilities. The testing will include tasks on logical and abstract thinking and creative tasks that require unconventional approaches.

Introduction of techniques: In the experimental group, students were taught using the proposed methods and teaching strategies (project activities, problem-based learning, use of ICT, active learning, group work, individual research projects, visualisation methods, feedback and support). In the control group, students were taught according to the traditional programme without applying the proposed methods. The experiment was conducted over one academic year, during which the student's progress was regularly monitored.

Data collection methods:

- *Questionnaires and interviews.* Conducting questionnaires and interviews with students and teachers to collect data on the perception and effectiveness of the methods.
- *Observation.* Systematic observation of the learning process in both groups to assess student engagement and activity.
- *Testing.* Conducting intermediate and final tests to assess changes in students' creative abilities.

Data analysis:

- *Comparative analysis of the test results of the experimental and control groups to identify differences in the level of development of creative abilities.*
- *Analysis of questionnaires and interviews to determine the perception of students and teachers of the proposed methods.*
- *Evaluation of observations to identify changes in students'* learning activity and engagement.

It is worth considering evaluating the experimental study's results. The table presents the analysis of the data obtained.

Appendices 2 and 3 present examples of tests, questionnaires, and interviews. The following indicators were obtained to assess the effectiveness of the proposed methods for developing students' creative abilities. These indicators are measured before the experiment starts and after it is completed. The table of resulting indicators is presented below.

Table 2. Evaluating the Results of an Experimental Study: Analysing the Data Acquisition

The key aspect	Features
Test results	*Initial testing:* The average scores of the initial test showed similar levels of creativity in the experimental and control groups. *Intermediate testing:* The experimental group showed significant progress in creative and logical thinking tasks compared to the control group. *Final test:* The experimental group demonstrated significantly higher results on all indicators of creative abilities, including abstract thinking, creativity and the ability to solve problems outside the box.
Questionnaires and interviews	*Students:* Most students in the experimental group reported that the new methods made lessons more exciting and engaging. They felt more motivated and engaged in the learning process. *Teachers:* Teachers reported an increase in student engagement and interest and improvements in their students' critical and creative thinking skills.
Observations	*Experimental group:* Observations have shown that students in the experimental group are more active in class, often offering non-standard solutions to problems and with great enthusiasm. *Control group:* In the control group, students demonstrated less engagement and activity and were less likely to offer creative solutions.

Source: Compiled by the author

Table 3. The experimental study's results

Indicator	Before the experiment (average score)	After the experiment (average score)
Creative thinking level	3.2	4.5
Analytical skills level	3.4	4.7
Logical thinking level	3.6	4.6
Abstract thinking level	3.3	4.4
Motivation to learn mathematics	3.1	4.8
Involvement in the learning process	3.5	4.9
Group work skills	3.0	4.6
ICT use in learning	2.8	4.5

Source: Compiled by the author

Indicators description:

- creative thinking: the performance of tasks requiring a non-standard approach and original solutions is evaluated;
- analytical skills: the performance of logical and analytical tasks is estimated;
- logical thinking: logical thinking is measured in solving problems;

- abstract thinking: the performance of tasks requiring abstract and spatial thinking is assessed;
- motivation to learn mathematics: evaluated based on a questionnaire about students' interest and motivation to learn mathematics;
- involvement in the learning process: assessed by observing students' activity and participation in class;
- group work skills: estimated by the ability of students to work effectively in teams and cooperate;
- ICT use in learning: assessed by the level of applying Information and Communication Technologies in the learning process.

After implementing the proposed mathematics teaching methods, these indicators demonstrate significant improvements in various aspects of student's creative abilities and learning activities.

The proposed mathematics teaching methods (project activities, problem-based learning, use of ICT, active learning, etc.) significantly contribute to the development of students' creative abilities. The experimental group showed higher results compared to the control group. Students taught using the new methods exhibited greater motivation and engagement in learning. Interactive and creative tasks made learning more exciting.

Students in the experimental group improved their critical thinking skills. They learned to analyse and evaluate different problem-solving approaches, an essential aspect of the creative process. Group work and collaboration fostered the development of communicative and creative skills. Students learned to exchange ideas and jointly find solutions to complex problems.

The research confirmed that integrating modern methods and strategies for teaching mathematics significantly promotes the development of students' creative abilities. These methods improve academic performance and increase interest and motivation for learning, developing skills necessary for successful academic and professional activities. Based on the research results, it is recommended that the proposed methods be implemented in the educational process to improve the quality of education and develop students' creative potential.

The following recommendations are proposed for teachers to implement effective mathematics teaching methods to develop students' creative abilities.

Integration of project activities:

1. *Project tasks.* Include projects in the curriculum that require the application of mathematical knowledge to solve real-world problems, such as projects on building modelling, environmental research, or data analysis.
2. *Examples of successful projects.* Share examples of successful projects from previous years to inspire creativity and show real-world applications of mathematical knowledge.

Use of problem-based learning:

1. *Problem tasks.* Regularly offer students tasks that require unconventional approaches and creative solutions. These can be open-ended tasks where students choose their method of solving them.
2. *Discussions and debates.* Organise discussions and debates on complex mathematical issues, encouraging students to analyse different points of view and argue their solutions.

Implementation of Information and Communication Technologies (ICT):

1. *Interactive programs.* Use programs like GeoGebra, Desmos, and other tools to visualise mathematical concepts and models. This helps students better understand and visualise complex tasks.

2. *Online resources.* Provide students with access to online courses, webinars, and interactive tasks to expand their opportunities for independent study and application of mathematical knowledge.

Active learning and game-based methods:

1. *Mathematical games.* Include mathematical games and quests in the learning process, making education exciting and engaging. For example, mathematical puzzles, competitions, and quizzes.
2. *Contests and Olympiads.* Organise mathematical contests and Olympiads, encouraging students to develop original solutions and improve their knowledge.

Organisation of Group Work:

1. *Work in Small Groups:* Divide students into small groups for joint tasks and project-solving. This promotes the development of communicative and creative skills.
2. *Cooperative Learning:* Apply cooperative learning methods in which students work together on common tasks, exchange ideas, and help each other.

Individual Research Projects:

1. *Research Tasks:* Encourage students to conduct independent research on mathematical topics that interest them. This can include data analysis, process modelling, or developing new theories.
2. *Scientific Papers:* Encourage students to write scientific papers and articles and present their research at school and extracurricular scientific conferences.

Methods of Visualisation and Spatial Thinking:

1. *Graphical Tasks:* Use tasks that require building graphs, diagrams, and other visual representations. This helps students better understand and analyse mathematical concepts.
2. *Modelling and Drawing:* Include tasks on modelling and drawing in the learning process, which develop spatial imagination and creative thinking.

Feedback and Support:

1. *Individual Consultations:* Provide students regular feedback and individual consultations to support and encourage their creative efforts.
2. *Achievement Portfolios:* Introduce portfolios where students can record their achievements and progress in solving creative tasks. This helps them see their successes and motivates further development.

Implementing the proposed methods and strategies for teaching mathematics fosters a stimulating and supportive environment that develops students' creative potential. These methods not only enhance academic performance but also develop skills necessary for successful academic and professional activities.

5. Discussion

The results of our study showed that geometric tasks, in combination with problem-based learning methods, project activities, group work, and information and communication technologies (ICT), significantly contribute to the development of students' creative abilities. These data confirm that integrating creative and analytical approaches in teaching mathematics effectively stimulates students' creative thinking and analytical skills.

Our conclusions are consistent with those of other authors, such as Dash and Panda (2021) and Khalid et al. (2020), who also found that problem-based learning and project activities significantly increase students' motivation and interest in studying mathematics. However, unlike some studies, we paid special attention to geometric tasks that promote the development of spatial thinking and visualisation, which is a unique contribution to our research.

The study's main results support the hypothesis that integrating different methods of teaching mathematics contributes to the development of students' creative abilities. Compared to the control group, our experimental group showed significant improvements in levels of creative thinking, logical thinking, and spatial thinking. These results indicate the need to reconsider traditional mathematics teaching methods in favour of more creative and interactive approaches.

Comparing our results with those of other researchers, such as Telegina et al. (2019) and Sari et al. (2018), we can note that project activities and the use of ICT indeed have a positive impact on the learning process. At the same time, our study emphasises the importance of integrating these methods with geometric tasks, which is often overlooked in other works.

Possible explanations for our results are that creative tasks and projects require students' active participation, analysis, and search for original solutions, which contributes to the development of their creative potential. Discrepancies with some studies may be related to differences in methodologies and research contexts.

The limitations of our study include the time frame of the experiment and the limited number of participants, which may affect the generalisation of the results. For a more accurate assessment of the effectiveness of the proposed methods, further research with a more significant number of participants and over a more extended period is required. The practical application of our results lies in recommendations for teachers to implement creative and interactive methods in teaching mathematics. This will create a stimulating and supportive learning environment that will promote the development of student's creative abilities and critical thinking, which is essential for their successful learning and professional activities.

6. Conclusion

The study showed that the use of modern methods of teaching mathematics, such as project activities, problem-based learning, and the integration of information and communication technologies, effectively contributes to developing students' creative abilities. The experimental group demonstrated a significant increase in creative thinking and analytical abilities compared to the control group. The results exceeded expectations, showing improvements in academic performance and increased motivation and engagement of students in the learning process. The novelty of the obtained data lies in the proven positive impact of a comprehensive approach to teaching the development of critical cognitive and creative skills. The practical significance of the results is confirmed by the potential for widespread implementation of the proposed methods in school curricula, which could enhance the quality of mathematics education. However, the study also faced some limitations, such as the time frame of the experiment and the limited number of participants, which may require further research to confirm the obtained data on a larger sample.

In the context of further research, it is recommended to continue working in this direction, paying particular attention to the long-term effects of applying new methods and their impact on developing other skills, such as critical thinking and the ability to learn independently. Implementing these methods in teaching practice also requires adaptation and the development of additional teaching materials to support students' creative activity.

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Appendix 1

Example 1a.
Consider a similar situation with isogonals AX and AY (Figure 5).

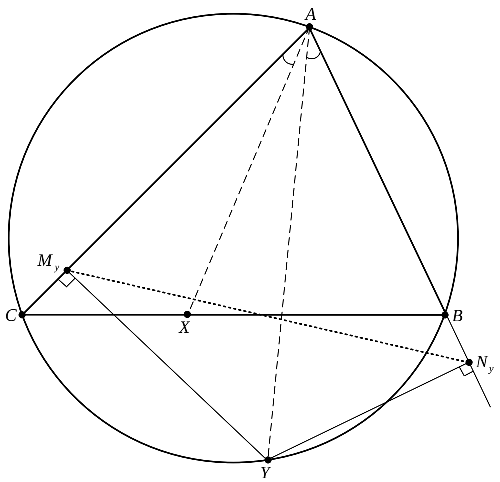

Figure 1

From the point Y belonging to the described circle around the triangle ABC, we drop perpendiculars YM_Y and YN_Y to the sides AC and AB of the triangle, ABC respectively.
In the following, the segments YM_Y and YN_Y will be called pedals of the point Y or bipeds of the point Y.
A circle with a diameter AY can be described around a quadrilateral AM_YYN_Y (Figure 2).
So,
$M_YN_Y = AY \sin \angle BAC.$ (Δ)
Form a quadrilateral AM_YXN_Y and prove that its area is equal to the area of triangle ABC.

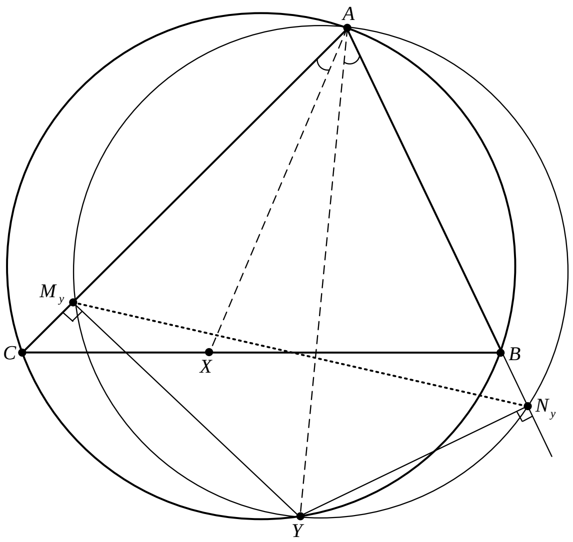

Figure 2

Otherwise, to prove that $S_{AM_YXN_Y} = S_{ABC}$.
Proof. The area of a quadrilateral AM_YXN_Y has the general form (Figure 3):
$S_{AM_YXN_Y} = \frac{1}{2} AX * M_YN_Y * \angle AEN_Y$,
where $AX \cap M_YN_Y = E$.
According to Figure 6: $AM_YN_Y = \angle AYN_Y$, because they are based on the same arc$\angle CAX = \angle BAY$ (by construction).
It follows that triangles AM_YE andAYN_Y are similar, and therefore,
$\angle AM_YE = AN_YY = 90°$.
Then,
$S_{AM_YXN_Y} = \frac{1}{2} AX * M_YN_Y * \sin 90° = \frac{1}{2} AX * M_YN_Y$. (ΔΔ)

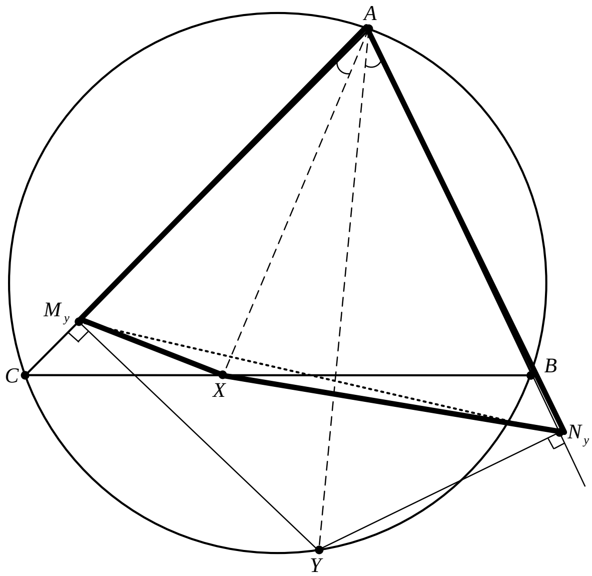

Figure 3

Applying the formula (Δ), we write (ΔΔ) differently:
$S_{AM_YXN_Y} = \frac{1}{2} AX * AY * \sin \angle BAC$. (ΔΔΔ).
Formula (***) makes it possible to write the area differently:
$S_{AM_YXN_Y} = \frac{1}{2} AC * AD * \sin \angle BAC = S_{ABC}$.
$S_{AM_YXN_Y} = S_{ABC}$. (2)
Proven.

We obtained and proved *the second equivalence* of the quadrilateral and triangle (2) using the isogonality of the lines AX and AY.

Appendix 2

Test to assess students' creative abilities

Part 1: Logical Thinking

Task 1: You have three boxes. One box contains only apples, another contains only oranges, and the third contains apples and oranges. The boxes are mislabeled. How can you determine what is in each box by opening only one box without looking into the others?

Task 2: There are three switches in a room. One turns on a light bulb in another room; the other two do nothing. You can enter the room with the light bulb only once. How can you determine which switch controls the light bulb?

Part 2: Abstract Thinking

Task 3: Imagine you are on another planet with a different measurement system. Draw what a clock would look like on this planet if there were 30 hours in a day and 50 minutes in an hour.

Task 4: Draw a 3D object that does not actually exist but could exist. Describe its functions and purpose.

Part 3: Creative Thinking

Task 5: Invent a new mathematical game to help students understand basic geometric shapes better. Describe the rules of the game and its goal.

Task 6: You are an architect. Create a design for a building that is both functional and artistically appealing. Sketch the building and describe which mathematical concepts you used in its creation.

Part 4: Spatial Thinking

Task 7: Draw the net of a cube where each face has different colours. Indicate the colours on the net and explain how you will fold the cube from this net.

Task 8: You need to build a bridge over a river. Draw a plan for the bridge, considering it must withstand a specific load and be wind-resistant. Write the mathematical calculations you will perform for the bridge design.

Part 5: Problem-Based Learning

Task 9: You are a scientist studying climate change. Use mathematical models to predict how the temperature on the planet will change over the next 50 years. Describe what data you will need and how you will analyse it.

Task 10: You are an entrepreneur who wants to open a café in your city. Calculate how many customers you must serve daily to make your business profitable. Consider rent, employee salaries, the cost of products, and other expenses.

These tasks aim to test various aspects of students' creative abilities, including logical, abstract, creative, and spatial thinking and problem-solving skills.

Appendix 3

Questionnaire and Interview to Assess Students' Creative Abilities

QUESTIONNAIRE FOR STUDENTS

1. Questions about perceiving the learning process:
How often do you encounter tasks that require an unconventional approach in your maths lessons?
☐ Very often
☐ Often
☐ Sometimes
☐ Rarely
☐ Never
How interested do you find such tasks?
☐ Very interesting
☐ Interesting
☐ Neutral
☐ Not very interesting
☐ Not at all interesting

2. Questions about motivation and engagement:
How much do you enjoy working in groups on mathematical projects?
☐ I like it a lot
☐ I like it
☐ Neutral
☐ Not very much
☐ I do not like it at all
Do you think tasks requiring a creative approach help you understand mathematics better?
☐ Yes, they help a lot
☐ Helpful
☐ Not sure
☐ Not very helpful
☐ Not at all helpful

3. Questions about teaching methods:
Which teaching methods help you to develop creative thinking the most? (multiple choices)
☐ Project tasks
☐ Problem tasks
☐ Group work
☐ ICT use (interactive programmes)
☐ Mathematical games
Would you like to see more creative tasks in maths lessons?
☐ Yes
☐ No
☐ Not sure

POSSIBLE QUESTIONS FOR STUDENT INTERVIEW

1. Questions about personal experience:
Can you give an example of a mathematics task you remember that required a creative approach? How did you solve it?
Have your problem-solving skills improved after the introduction of the new teaching methods? If so, how?

2. Questions about the perception of new methods:
How do you evaluate working in groups on mathematical projects? What do you like or dislike about this process?
Which tasks or projects do you remember the most and why?

3. Questions about motivation and interest:
How much more interesting has it become for you to study mathematics with the introduction of new methods and tasks? Why?
Do you feel more confident solving complex tasks after using the new methods?

POSSIBLE QUESTIONS FOR STUDENT INTERVIEW

1. Questions for observing students:
What changes have you noticed in students' activity and engagement after the introduction of the new teaching methods?
How do students respond to tasks that require a creative approach? Have they become more interested and motivated?

2. Questions on the effectiveness of methods:
Which proposed methods (project activities, problem-based learning, ICT, etc.) have effectively developed students' creative abilities?
Can you provide specific examples of student successes made possible by the new methods?

3. Questions about difficulties and improvements:
Have you encountered any difficulties in implementing the new methods? If so, what were they?
What improvements or changes to the methods would you suggest to enhance their effectiveness?

These questionnaires and interview questions will help gather qualitative data on the perceptions, motivations, and effectiveness of the proposed methods of teaching mathematics that aim to develop students' creative abilities.