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THE APPLICATION OF BOX-BEHNKEN METHOD TO OPTIMIZE THE DESIGN OF EWMA CHART FOR AUTOCORRELATED PROCESSES

Abstract: *This research aims to evaluate the performance of the exponentially weighted moving average (EWMA) control chart under the situation that the observations are autocorrelated. Three autoregressive integrated moving average (ARIMA) models, AR (1), ARMA (1, 1) and IMA (1, 1), and a step function were utilized to characterize the process model. The autocorrelated observations were monitored by the exponentially weighted moving average (EWMA) chart and the average run length (ARL) was used as the performance index. A response surface method, Box-Behnken design, was utilized to carry out the optimal design of the EWMA parameters, λ and L, while the robustness of the control chart was still maintained when there was no shift in the process. The empirical results show that the autocorrelation has a significant effect on the value of the ARL, i.e., the ability to detect a special cause and the occurrence frequency of a false alarm. Another important finding is that, under the autocorrelated situation (both stationary and non-stationary), the control limits of the EWMA chart should be narrowed down to L = 2 for the best performance. On the other hand, the value of λ does not seem to have a significant effect on the ARL except only when the observation follows ARMA (1, 1). Moreover, the results also reveal that the size of a shift will impact the detection sensitivity of the EWMA to a shift only when the process is stationary. According to the study, if the EWMA chart utilized under the autocorrelated environment is appropriately designed, the practitioners on the shop floor will have a state of the art guidelines for achieving the highest possible performance when deploying the EWMA chart.*

Keywords: *autocorrelation, autoregressive integrated moving average (ARIMA), average run length (ARL), box-behnken design, exponentially weighted moving average (EWMA) chart*

1. Introduction

Statistical process control (SPC) is a methodology used for monitoring and reducing the variation in the manufacturing process and one of the most important tools of SPC are the different types of control charts. Generally, SPC works under the assumption that the observed data is independent. However, because of the advanced measurement technology, shortened sampling interval and the nature of processes, especially in the continuous processes, e.g., chemical processes, the independence of each observation has been violated in many scenarios. The lack of independence among each sample always comes up in the form of a serial autocorrelation. Therefore, the consequence is that a control chart signal fault alarms more often or does not signal when there is a shift. This behavior of process outputs has significantly deteriorated the performance of control charts and leads to the extensive studies regarding the SPC improvement for autocorrelated processes.

2. Literature review

The first step leading to the successful characterization of any standard charts is the capability to simulate the different types of autocorrelated processes. Under the normal and uncorrelated conditions, the process model has a fixed mean (μ), and the fluctuation around the mean is the result of a white noise (at). However, when observations are correlated, the correlation structure and a drift in the mean can be characterized by disturbances. If process observations vary around a fixed mean and have a constant variance, this type of variability is called the stationary behavior. Otherwise, the behaviour is non-stationary. (MacGregor, 1998) indicated that there are two types of disturbances, deterministic and stochastic disturbances. Stochastic disturbances are random and might be stationary or non-stationary so it is the main source of autocorrelation in the data. On the other hand, deterministic disturbances are a step shift or ramp in the process mean. (Box *et al.*, 1976) introduced a stochastic difference equation that can model stochastic disturbances. This equation is used to forecast one-step ahead disturbances, according to the data characteristics of stationary or non-stationary. Normally, it is expressed in the form of an autoregressive integrated moving average model, ARIMA, as shown in equation (1).

$$\Delta_d Y_t = \mu + \phi_1 \Delta_d Y_{t-1} + \phi_2 \Delta_d Y_{t-2} + \ldots + \phi_p \Delta_d Y_{t-p} + a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q} \quad (1)$$

The ARIMA (p, d, q) model indicates p as the order of the autoregressive part, d as the amount of difference and q as the order of the moving average part. As recommended by Box *et al*. (1976), ARIMA (1, 0, 0) or AR (1) and ARIMA (1, 0, 1) or ARMA (1, 1) are likely to be the most suitable models to represent stationary processes while ARIMA (0, 1, 1) or IMA (1, 1) is the appreciated choice for non-stationary process. Therefore, several authors (Noorossana et al., 2003; Apley, 2012; Hwang *et al*., 2008; MacCarthy and Wasusri, 2001; Jiang *et al*., 2000) utilized the ARIMA model to simulate both stationary and non-stationary processes. Moreover, the performance of the traditional Shewhart chart was assessed by Wardel *et al*. (1992) when the tested data was modeled by ARIMA (1, 1). According to their work, the average run length (ARL) was used to measure the robustness of the designated chart. (Schmid, 1995) had computed the exact ARL of the classical Shewhart chart under the situations that the process was autocorrelated and followed the AR (1) model. The exploration of how EWMA chart performed was conducted by Schmid and Schone (1997) in the scenario that the observations were stationary. The autocorrelated scenarios were simulated by

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using the Monte Carlo technique while the ARL was determined from the simulation.

Moreover, the performance of each standard chart is also assessed by benchmarking with each other. One of these works is the performance comparison of $\overline{X}$ and EWMA chart by English *et al*. (2000). The objective is to quantify the ability of each chart to detect a special cause when the data is autocorrelated. Wardell *et al*. (1992) had compared the performance of the traditional and EWMA chart when the observations followed the ARMA (1, 1) model. The inside details regarding the run length was also studied by Wardell *et al*. (1994) when the observations was modeled by the ARMA (1, 1). According to the study, the ARL and its standard deviation were calculated in order to determine the probability to detect an assignable cause at the early stage. The performance of the EWMA and CUSUM charts was numerically compared by VanBrackle and Reynolds (1997). For the comparison, the ARL of these two charts were computed when the process followed the AR (1) model. For multivariate times series, Kramer and Schmid (1997) studied the capability of the EWMA chart for monitoring multivariate time series which were also autocorrelated and simulated by the AR (1) model.

Another way to improve the performance of standard charts is to filter the correlated data with the ARIMA model and the residual from the filter is monitored by the designated control chart. This technique was used by MacCarthy and Wasusri (2002) and Lu and Reynolds (1999) to monitor the residual based data with the EWMA chart. Superville and Adams (1994) had utilized the ARIMA model to forecast the autocorrelated observations and the predicting residual was monitored by the CUSUM and EWMA charts. The step function was used to represent a special cause.

In some cases, the actual data is utilized instead of the simulated data and these sets of data are acquired from different sources. For example, Vargas *et al*. (2004) had benchmarked the performance of two different charts (CUSUM and EWMA) to monitor the autocorrelated data acquired from a production process. Similarly, Winkel and Zhang (2004) compared the performance of two different control charts for monitoring the biochemical quality data which was autocorrelated. The application of SPC and autocorrelated data is not limited to only the industrial data but the information technology data as well. For example, Ye *et al*. (2003) applied the EWMA chart to detect the analogous changes in the event intensity for intrusion detection while the data was correlated and simulated by deploying the AR (1) model. According to the literature, the EWMA is one of the most frequently used control charts to monitor the autocorrelated processes because of its sensitivity to a small shift. Moreover, its design is flexible since it allows the practitioners to select the suitable parameters to achieve the highest possible performance. However, there are a limited number of studies regarding the optimal design of the EWMA chart towards the autocorrelated observations. Moreover, most researches in the area do focus only on the specific process (stationary or non-stationary case) of the autocorrelation instead of the holistic point of view (both cases) so it is difficult to select the optimal value for designing the EWMA chart. In this research, a response surface method, Box-Behnken approach, is utilized as a mean to analyze and optimize the empirical results effectively and to achieve the optimal values of the EWMA parameters, λ and L. For simplicity, this study concentrates on the actual value of the observations since the residual monitoring seems to be difficult for practitioners to use. As a result, this research will fulfill the gap and highlight the suitable setting of EWMA's λ and L under the autocorrelation scenario.

3. Innovations

The basis of the analysis in this paper is a mathematical model used to study the effects of the process autocorrelation on the performance of the EWMA chart. Process disturbances are controlled by adjusting the level of autocorrelation in the form of the ARIMA parameters while a special cause is simulated by a step function. The autocorrelated process in this study is a continuous process with a quality characteristic, represented by Y. The evaluation of the EWMA chart performance is measured by considering the average run length (ARL) which is the average number of points plotted before a point indicated the out-of-control state. The observation of a process is considered from period 1 to 550 (t = 1, 2, 3,…, 550) and the process output (Y_{t+1}) is equal to

$$Y_{t+1} = T + N_{t+1} + \delta(t) \qquad (2)$$

The sources of the autocorrelation are process disturbances, characterized by the autoregressive integrated moving average model, AR (1), ARMA (1, 1) and IMA (1, 1), as shown in equation (3), (4) and (5):

$$N_{t+1} = \phi N_t + a_{t+1} \qquad (3)$$

$$N_{t+1} = \phi N_t + a_{t+1} - \theta a_t \qquad (4)$$

$$N_{t+1} = N_t + a_{t+1} - \theta a_t \qquad (5)$$

where N_{t+1}, N_t are the disturbances at time t+1 and t respectively, a_{t+1}, a_t are the random errors at time t+1 and t respectively, ϕ is the autoregressive (AR) parameter and θ is the moving average (MA) parameter. The values of ϕ and θ are between -1 and 1. Afterwards, an EWMA chart with the following control limits as:

$$\left.\begin{aligned} UCL &= \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]} \\ CL &= \mu_0 \\ LCL &= \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]} \end{aligned}\right\} \qquad (6)$$

where μ0 is the average of preliminary data, L is the width of control limits and λ is the weight assigned to the observation, is utilized to monitor the autocorrelated observations. To simulate a special cause, a shift of size δ0 which is in the form of a step function is applied into a process at time t =50 as:

$$\delta(t) = \begin{cases} 0; t < t_{50} \\ \delta_0; t \geq t_{50} \end{cases} \qquad (7)$$

where δ0 is the magnitude of a shift and t0 is the time that a shift occurs.

4. Design of experiment

To statistically optimize the monitoring procedure, a response surface method, Box-Behnken design, was utilized to design an EWMA chart when the observations are correlated. Surface experiments are performed to fit either a first order model (linear function) or a second order model to the observations. The advantage of the Box-Behnken technique is that it does not include any points at the vertices of the cubic region and the resulting design is still rotatable. Fig. 1 shows the graphic example of the Box-Behnken design for three factors.

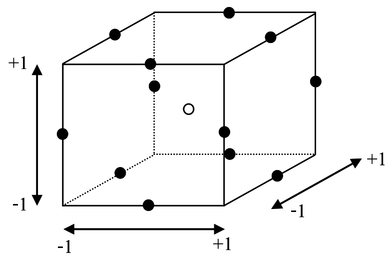

Figure 1. Box-Behnken Design for Three Factors

Due to the study, all the potential factors, namely autocorrelation coefficients (ϕ and θ), shift size and the EWMA chart's parameters, λ and L, are investigated to quantify the effect and optimize their values. Afterwards, the values of λ and L are characterized in order to minimize the average run length (ARL) when an assignable cause does occur. Moreover, the robustness of an EWMA chart is also assessed when there is no shift.

5. EWMA characterization

The empirical study is based on the experiment relying on three process models: AR (1), ARMA and IMA (1, 1), which are widely used to simulate the autocorrelation process (both stationary and non-stationary). Regarding the simulation, each run or treatment is composed of 10,000 iterations which are accomplished by using Palisade's @Risk® version 5.7. The random errors (at) in the disturbance model from each period follow the normal distribution with zero mean and a constant variance as: $a_t \sim N(0,1)$. The starting value (μ0) of the EWMA chart is equal to zero while the standard deviation of the chart is 1. According to the shift size, it is set at 1, 2, 3 and 4 and the Box-Behnken design of experiment is done using a statistical package, Design Expert® version 8.0.7.1, to analyze the effect of the autocorrelation and other factors on the response.

5.1 AR (1)

The AR (1) process is popularly deployed to characterize the stationary process. Four factors, ϕ, λ, L and shift size, are studied and shown in table 1 (the range of λ, L and shift are adopted from the study by Lucas and Saccucci (1990) while the Box-Behnken design matrix for the experiment is shown in table 2.

Table 1. Input factors and levels for AR (1) Case

Factor	-1	0	1
A (AR parameter; ϕ)	-1	0	1
B (λ)	0.05	0.15	0.25
C (Shift size)	0	2	4
D (L)	2	3	4

Table 2. Design Matrix for AR (1) case

Order	ϕ	λ	Shift	L	ARL
1	-1	0.05	2	3	8.78
2	1	0.05	2	3	1.757
3	-1	0.25	2	3	4.04
4	1	0.25	2	3	1.453
5	0	0.15	0	2	41.92
6	0	0.15	4	2	1.476
7	0	0.15	0	4	481.86
8	0	0.15	4	4	2.59
9	-1	0.15	2	2	2.52
10	1	0.15	2	2	1.236
11	-1	0.15	2	4	9.87
12	1	0.15	2	4	1.662
13	0	0.05	0	3	318.88
14	0	0.25	0	3	264.19
15	0	0.05	4	3	3.03
16	0	0.25	4	3	1.697
17	-1	0.15	0	3	50.12
18	1	0.15	0	3	2.49
19	-1	0.15	4	3	2.21
20	1	0.15	4	3	1.286
21	0	0.05	2	2	4.05
22	0	0.25	2	2	2.31
23	0	0.05	2	4	8.23
24	0	0.25	2	4	5.62
25	0	0.15	2	3	4.09
26	0	0.15	2	3	4.24
27	0	0.15	2	3	4.15

28	0	0.15	2	3	3.98
29	0	0.15	2	3	4.04

As shown in table 3, the statistical analysis was conducted in the form of the analysis of variance (ANOVA) and the inverse transformation was used to ensure that the residual does not violate the underlying i.i.d. assumptions. Due to the ANOVA, three factors, A (ϕ), C (shift) and D (L), along with the curvature (C2) has a significant effect on the value of ARL. In addition, it is interesting to note that the EWMA parameter, λ, is not the significant factor when the data follows the AR (1) model.

Table 3. ANOVA for AR (1) case

Source	SS	Df	MS	F	p-value
A-ϕ	0.395145	1	0.395145	30.2135	< 0.0001
C-Shift	0.879446	1	0.879446	67.24408	< 0.0001
D-L	0.106506	1	0.106506	8.143654	0.0088
C^2	0.073681	1	0.073681	5.633797	0.0260
Residual	0.313882	24	0.013078		
Total	1.768659	28			

After the model for analyzing the ARL in the AR (1) case was derived, the main effect plot was generated to quantify the influence of each significant factor. The Fig. 2 illustrates the relationship between ϕ and ARL when the values of the other factors were set at the mid-points (average) as: λ = 0.15, shift = 2 and L = 3.

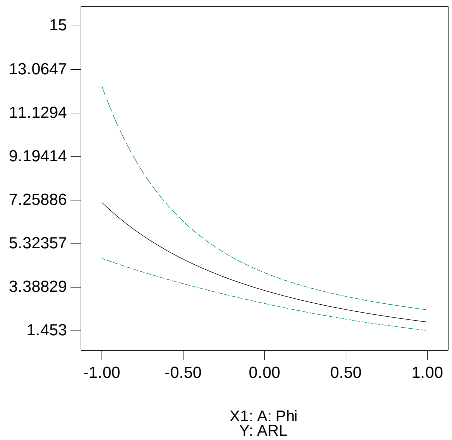

Figure 2. Main Effect Plot of ϕ

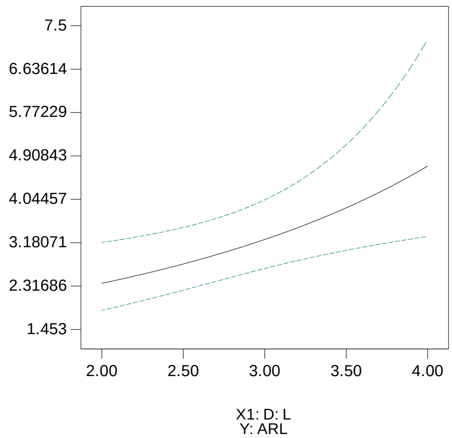

Figure 3. Main Effect Plot of L

According to Figure 2, the response surface analysis revealed that, for the AR (1) case, the more negative the ϕ is, the higher value the ARL is. Furthermore, due to Fig. 3, the only EWMA design factor to be concerned (the effect of other factors are averaged at: ϕ = 0, λ = 0.15 and shift = 2) is the multiple of sigma in the control limit (L) which should be assigned the value at 2 in order to minimize the ARL. However, another EWMA parameter, i.e., λ, is not significant so it can be set at any values between 2 and 4.

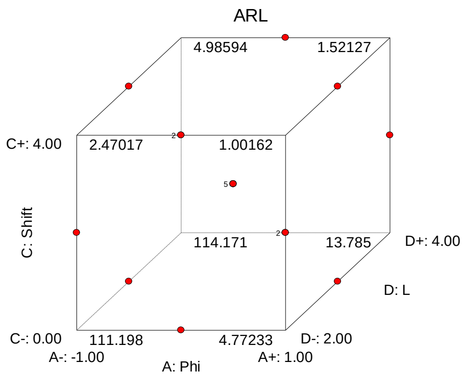

Figure 4. Cube Plot of A, C and D

According to the cube plot (Figure 4), the autoregressive coefficient (ϕ) has a significant effect on the in-control ARL (ARL0) when λ is averaged at 0.15. The ARL0 range is between 4.77233 (ϕ = 1) and 111.198 (ϕ = -1) instead of 500 when there is no autocorrelation.

5.2 ARMA (1, 1)

The number of factors to be considered in the ARMA (1, 1) case is similar to AR (1) case except that there were two parameters in the ARIMA model (ϕ and θ). The ARMA (1, 1) is another model widely used in the literature to characterize the stationary processes. For optimizing the ARL, the factors to be tested are listed in table 4.

Table 4. Input factors and levels for ARMA (1, 1)

Factor	-1	0	1
A (AR parameter; ϕ)	-1	0	1
B (MA parameter; θ)	-1	0	1
C (λ)	0.05	0.15	0.25
D (Shift size)	0	2	4
E (L)	2	3	4

All the above factors were tested for its influence on the ARL by deploying the Box-Behnken design, and the completed design matrix is illustrated in table 5.

Table 5. Design Matrix for ARMA (1, 1) Case

Order	ϕ	θ	λ	Shift	L	ARL
1	-1	-1	0.15	2	3	9.78
2	1	-1	0.15	2	3	1.25
3	-1	1	0.15	2	3	2.97
4	1	1	0.15	2	3	2.48
5	0	0	0.05	0	3	316.01
6	0	0	0.25	0	3	264.38
7	0	0	0.05	4	3	3.03
8	0	0	0.25	4	3	1.693
9	0	-1	0.15	2	2	2.87
10	0	1	0.15	2	2	2.51
11	0	-1	0.15	2	4	6.28
12	0	1	0.15	2	4	5.47
13	-1	0	0.05	2	3	8.75
14	1	0	0.05	2	3	1.724
15	-1	0	0.25	2	3	4.03
16	1	0	0.25	2	3	1.452
17	0	0	0.15	0	2	43.82
18	0	0	0.15	4	2	1.465
19	0	0	0.15	0	4	480.18
20	0	0	0.15	4	4	2.6
21	0	-1	0.05	2	3	6.43
22	0	1	0.05	2	3	5.8
23	0	-1	0.25	2	3	3.93
24	0	1	0.25	2	3	3.25
25	-1	0	0.15	0	3	49.43
26	1	0	0.15	0	3	2.46
27	-1	0	0.15	4	3	2.22
28	1	0	0.15	4	3	1.292
29	0	0	0.05	2	2	4.05
30	0	0	0.25	2	2	2.31
31	0	0	0.05	2	4	8.22
32	0	0	0.25	2	4	5.58
33	-1	0	0.15	2	2	2.5
34	1	0	0.15	2	2	1.237

35	-1	0	0.15	2	4	9.92
36	1	0	0.15	2	4	1.655
37	0	-1	0.15	0	3	22.1
38	0	1	0.15	0	3	500
39	0	-1	0.15	4	3	2.12
40	0	1	0.15	4	3	1.994
41	0	0	0.15	2	3	4.1
42	0	0	0.15	2	3	4.09
43	0	0	0.15	2	3	4.11
44	0	0	0.15	2	3	4.12
45	0	0	0.15	2	3	4.09
46	0	0	0.15	2	3	4.08

After running the analysis, the results shows that the inverse transformation is required for sustaining the residual assumptions. The ANOVA in table 6 points out that the following factors, A (ϕ), C (λ), D (shift), L and A2, are highly significant. For the analysis, the main effect plot of ϕ in Fig. 5 portrays that the relationship between ϕ and the ARL is non-linear. Moreover, the value of ARL seems to be low when ϕ is highly positive. On the other hand, when the value of ϕ falls in the negative range (from -1 to 0), the ARL is higher than the ones for ϕ in the positive range (from 0 to 1).

Table 6. ANOVA for ARMA (1, 1) Case

Source	SS	Df	MS	F	p-value
A-ϕ	0.677432	1	0.677432	137.6717	< 0.0001
C-λ	0.060211	1	0.060211	12.23647	0.0012
D-Shift	0.846273	1	0.846273	171.9844	< 0.0001
E-L	0.161167	1	0.161167	32.75326	< 0.0001
A^2	0.332763	1	0.332763	67.62609	< 0.0001
Residual	0.196825	40	0.004921		
Total	2.274671	45			

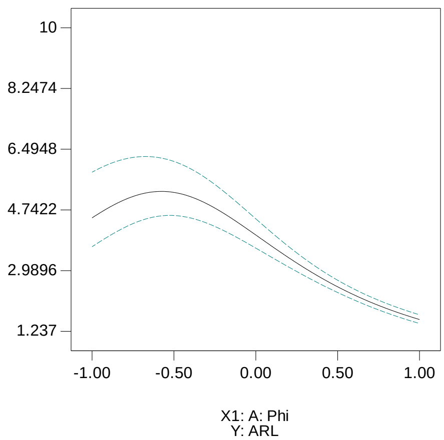

Figure 5. Main Effect Plot of ϕ

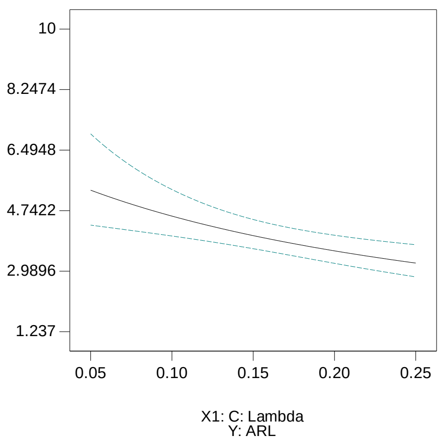

Figure 6. Main Effect Plot of λ

Additionally, the optimization was performed in order to determine the design of EWMA chart. Due to Fig. 6, the optimal values of λ to minimize the ARL should be set at the lowest one at 0.25 (other factors were set at the mid-points). For the optimal value of L, Fig. 7 shows that the width of control limit should be narrowed to the lowest possible value at 2.

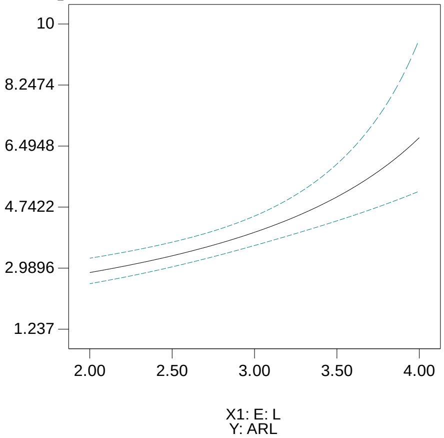

Figure 7. Main Effect Plot of L

5.3 IMA (1, 1)

According to the literature, the AR (1) and ARMA (1, 1) are the preferred models for characterizing the stationary processes. On the other hand, the IMA (1, 1) is the most popular model for representing the non-stationarity. The design matrix for the Box-Behnken method can be shown in table 7 while the results after the experiment are shown in table 8.

Table 7. Input Factors and Levels for IMA (1, 1) case

Factor	-1	0	1
A (MA parameter; θ)	-1	0	1
B (λ)	0.05	0.15	0.25
C (Shift size)	0	2	4
D (L)	2	3	4

Table 8. Design Matrix for IMA (1, 1) case

Order	θ	λ	Shift	L	ARL
1	-1	0.05	2	3	1.687
2	1	0.05	2	3	313.51
3	-1	0.25	2	3	1.478
4	1	0.25	2	3	262.13
5	0	0.15	0	2	1.698
6	0	0.15	4	2	1.685
7	0	0.15	0	4	3.5
8	0	0.15	4	4	3.49
9	-1	0.15	2	2	1.225
10	1	0.15	2	2	42.29
11	-1	0.15	2	4	1.706
12	1	0.15	2	4	478.49
13	0	0.05	0	3	3.31
14	0	0.25	0	3	2.69
15	0	0.05	4	3	3.31
16	0	0.25	4	3	2.66
17	-1	0.15	0	3	1.427
18	1	0.15	0	3	281.33
19	-1	0.15	4	3	1.408
20	1	0.15	4	3	280.76
21	0	0.05	2	2	2.16
22	0	0.25	2	2	1.782
23	0	0.05	2	4	4.75
24	0	0.25	2	4	3.93
25	0	0.15	2	3	2.52
26	0	0.15	2	3	2.44
27	0	0.15	2	3	2.4
28	0	0.15	2	3	2.52
29	0	0.15	2	3	2.47

Table 9. ANOVA for IMA (1, 1) Case

Source	SS	Df	MS	F	p-value
A-θ	1.690822	1	1.690822	1204.037	< 0.0001
D-L	0.11607	1	0.11607	82.65388	< 0.0001
A^2	0.202574	1	0.202574	144.253	< 0.0001
Residual	0.035107	25	0.001404		
Total	2.044573	28			

Before conducting the analysis of variance in table 9, the inverse square root transformation is applied to the response to avoid the violation of the residual assumptions. According to table 10, only two factors, A (θ), D (L) and A^2, have a significant effect on the ARL. When the non-stationary process follows the IMA (1, 1) and an EWMA chart is utilized to monitor the process, the MA parameter (θ) seems to have a significant effect on the ARL (Fig. 8). The ARL tends to be high when the value of θ is highly positive (Fig. 9). For the EWMA parameter, i.e., L in this case, the optimal value for L should be set at 2 since it leads to the minimization of ARL.

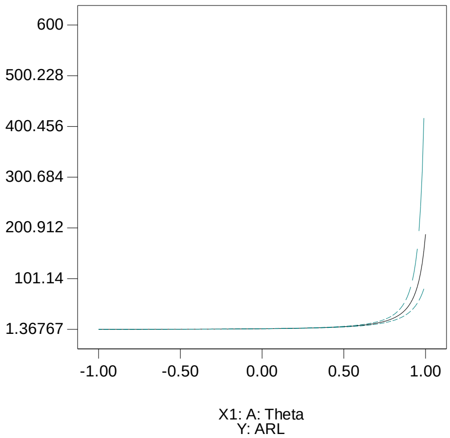

Figure 8. Main Effect Plot of θ

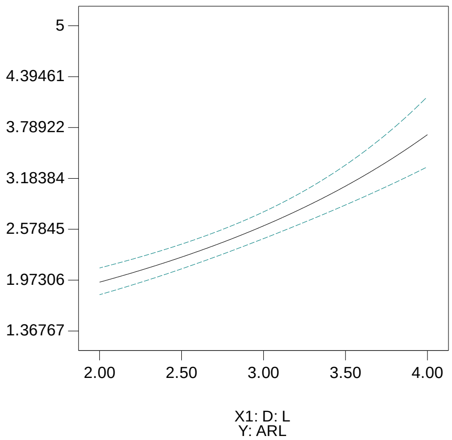

Figure 9. Main Effect Plot of L

6. Conclusion

This research focuses on the optimal design of EWMA chart on stationary and non-stationary processes based on three different models, AR (1), ARMA (1, 1) and IMA (1, 1), with the implementation of the Box-Behnken method to analyze the data. The resultant analysis is concluded as follows:

1. According to all cases of autocorrelation, the optimal value of EWMA parameter, L, should be narrower than the traditional ones in the literature and should be set at 2.
2. Only for the processes following ARMA (1, 1), the design parameter of EWMA, λ, is significant and should be set at 0.25.
3. Obviously, the autocorrelation coefficients do have a significant effect on the capability to detect an assignable cause (ϕ for stationary case and θ for non-stationary case) in the autocorrelated environment.
4. According to the empirical results, the existence of a shift in the process will

affect the ARL only when the observations are stationary. On the other hand, the special cause does not have a significant impact under the non-stationary situation.

In conclusion, the appropriate design of EWMA will lead to the better performance of a control chart to detect a shift resulted from a special cause in the autocorrelated processes. The different categories of stationarity need a different chart design and it will facilitate the application of practitioners when the process is autocorrelated.

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