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SOFTWARE APPLICATION FOR MARKET MANAGEMENT FORECASTING: A MARKOV CHAIN APPROACH

Abstract: Markov chain is a widely used method for future trends forecasting. The aim of this paper is to present the development and functionality of a software application for solving the problem of sales and number of customers forecasting by using the Markov chain method. On a concrete company whose main activity is the distribution and sale of expensive cosmetic products, solutions to the forecasting problem and the functionality of the developed software application are presented. Based on the information on sales of certain products or the number of customers in the observed market, it is possible to devise adequate business strategies to increase its profitability and competitiveness in the market, which greatly facilitates the use of such software application.

Keywords: Markov chain, Software application, Market management forecasting

1. Introduction

Nowadays, business is dynamic and extremely competitive and requires quick and quality decisions in order to achieve satisfactory business results. In order to make the best possible decisions, various methods are used to forecast the movement of certain phenomena in the future. There are several methods of forecasting in the literature, from simpler to more complex. The Markov chain method is one of the forecasting methods widely used in various fields. Markov chains are a kind of stochastic processes in discrete time where every future state in that process depends only on the present state, not on previous states. The purpose of using Markov chains is to find a steady-state which the phenomenon will take over as its permanent state after a certain time.

By using Markov chains, sales and the number of customers forecasting in the observed market can be predicted. Therefore, the aim of this paper is to present the development and functionality of a software application for solving sales problems forecasting and number of customers forecasting by using Markov chains in order to facilitate the company to make adequate business decisions. With the help of a software application based on Markov chains, users will be able to determine the growth/decline in sales of certain products or the reduction/increase in the number of customers. The advantage of using Markov chains in business is to monitor changes that occur in the market and adjust the company's business strategy to these changes. Based on the information obtained through Markov chains, companies can focus on the most profitable and best-selling products, as well as increase the chances of improving those products that are falling sales or make a final decision to withdraw such products from the sales range.

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The paper is structured as follows: after the introduction, a review of the literature on the application of Markov chains in the field of market management forecasting is given. The third section describes the methodological framework of the Markov chain model. The fourth section describes the problem of sales and the number of customers forecasting in a particular sales company from the real system. The fifth section presents the development and functionalities of a software application for solving forecasting problems based on the Markov chain. The last chapter is a conclusion with recommendations for future research.

2. Markov chain application in market management forecasting

Markov chains have a wide application in predicting the movement of various phenomena. In the continuation of this paper, a review of the literature on the application of Markov chains is given, with an emphasis on market management forecasting.

As one of the most common methods for predicting market share in the economy, Markov chain were used in estimating the future market shares of Romanian banking institutions (Kovacs, 2018). Banks are ranked each year based on net asset status to monitor the performance and outlook of the banking sector. This ranking is very important for economic entities regardless of the volume of business, for clients and potential clients of banks. On the example of the telecommunications industry, Chan (2015) tested four mathematical models for the same market share problem based on different underlying assumptions. The analysed models are: a homogeneous Markov model, a time-varying Markov model, a new extended time-varying Markov model, and a novel non-Markov model. The analysis of the results indicates that all models can be used to model and predict market share with an emphasis on the need to have a deep understanding of industry, market conditions and trends. To analyse the share of the mobile phone market, the authors applied Markov processes and other mathematical methods. The aim of the analysis was to establish and improve the transition probability matrix to overcome the shortcomings of traditional prediction methods and to adapt to the lack of Markov chains. The result shows that the state transition matrix can be a complex and unstable market for dynamic adjustment to meet the needs of market positioning. Based on Markov decision process, Chan, Lenard and Mills (2013) developed a model for evaluating business strategies for the telecommunications industry. The model enables mobile service providers to make decisions and predict market shares and financial rewards.

Given unpredictable markets, enterprise management strategies are extremely important. In order to improve the level of business management and establish the correct business philosophy, Tian, Zhang and Li (2016) proposed Markov chain method in market management forecasting. In order to make better decisions and achieve higher profits, Markov chain models in China's market economy have been developed to predict the percentage of market share and profit of producers (Zhou, 2013). The Markov chain method forecasted the entry and exit of Polish companies in certain business sectors, as well as the migration of companies through sectors (Nehrebecka, 2011). The average remaining life and the average age of the company were also calculated. Analysis of the Markov chain results indicates the stability of companies, low level of migration between sectors and relatively short life expectancy of a company. In order to make the best possible decision, it is important to recognize the variability of dynamic and unstable economic variables. Waller, Adablah, and Kester (2019) proposed a model for

predicting economic variables for making critical decisions by using Markov chains.

The forecast of market shares and number of tourists were analysed in Sour (2020). Based on data on the number of international arrivals in China, Spain, USA, France, Italy and Mexico, three models of Markov chains have been developed to estimate the probability of a tourist visiting one of these destinations. In the long run, China will lead with the largest market share. To provide the necessary information to government policy makers, travel companies, airlines, and researchers, Choi, Mok and Han (2011) developed a Markov chain model to predict international tourism flows.

Markov chains are also used in human potentials management to predict employee behaviour in the future. Belhaj and Tkiouat (2013) presented a model of employee prediction in a time-dependent hierarchical human resources system that divides employees into homogeneous subgroups. Markov's model has also been used in military planning in the United States (Zais & Zhang, 2016), while human resource forecasting for an IT company for a three-year period is presented in Žajdela Hrustek, Keček, Polgar (2020).

In some studies, the Markov chain methodology is supplemented by other methods. The embedded input-output model in Markov chain is a suitable methodology for understanding sector connectivity. The synergistic effects of the application of the input-output model and Markov chain contribute to the quality of cross-sectoral analysis and a more detailed analysis of financial flows (Keček, Dušak, Žajdela Hrustek, 2015; Duchin & Levine, 2010). Markov chains have rarely been used in the field of investment, while Markowitz's portfolio optimization model is one of the most popular concepts of portfolio theory. Škrinjarić and Šoštarić (2014) for the first time analysed the complementarity of Markov chains and the Markowitz model in an attempt to find an efficient portfolio.

They concluded that the Markowitz model is better at finding an efficient portfolio and that Markov chains with some modifications can be helpful in optimizing the Markowitz model.

3. Markov chain method

The Markov chain method is a statistical method for analysing the motion of a certain random variable with the purpose of predicting its motion in the future. It is based on transition probabilities and uses this probability to show transitions from one state to another. The basic mathematical aspects of Markov chains applied in this paper are given below. In more detailed about Markov's chains can be found in Winston and Goldberg (2004), Brooks et al. (2011), Brémaud (2020).

A sequence of random variables $(X_t, t \epsilon \mathbb{N}_0)$ with a state space $S = \{i, j, i_0, i_1, \dots, i_{t-1}\}$ is called Markov chain if it has Markovian property:

$$\mathbb{P}(X_{t+1} = j | X_t = i, X_{t-1} = i_{t-1}, \dots, X_0 = i_0) = \mathbb{P}(X_{t+1} = j | X_t = i), t \epsilon \mathbb{N}_0, \ i, j \epsilon S \quad (1)$$

In (1), $\mathbb{P}$ represents the conditional probability of transition from state at the moment t to state j at the moment $t + 1$. Thus, the conditional probability of the transition between state i and state j does not depend on the states that preceded those states. Also, the transition probability of the Markov chain from state i to state j does not depend on the moment at which the transition occurs, i.e. Markov chain is homogeneous in time:

$$p_{ij} = \mathbb{P}(X_{t+1} = j | X_t = i) = \mathbb{P}(X_1 = j | X_0 = i) \quad \text{for each} \quad i, j \epsilon S, t \epsilon \mathbb{N}_0 \quad (2)$$

The transition probability p_{ij} represents the probability of the chain transitioning from state i to state j in one step. Elements of the transition probability matrix:

$$P = [p_{ij}]_{i,j\epsilon S} \qquad (3)$$

are non-negative and the sum of the elements in each row of the transition probability matrix equals to one. Apart from the distribution of the transition from one state to another state in one-time step, the behaviour of the Markov chain is completely determined by the distribution of the random variable X_0, i.e. by the initial distribution of the chain:

$$p_i^{(0)} = \mathbb{P}(X_0 = i), i\epsilon S \qquad (4)$$

Probability (4) represents the probability that the process is in the i state at the initial moment of observation. The initial state vector is denoted by:

$$p^{(0)} = \left(p_1^{(0)}, p_2^{(0)}, \dots\right) \qquad (5)$$

and the state vector after n steps by:

$$p^{(n)} = \left(p_1^{(n)}, p_2^{(n)}, \dots\right) \qquad (6)$$

where $p_i^{(n)} = \mathbb{P}(X_n = i), i\epsilon S$ indicates the probability that the process will be at state i after n steps. $P^{(n)}$ denotes the transition probability matrix in n steps, where the probability of transition from state i to state j in n steps is equal to:

$$p_{ij}^{(n)} = \mathbb{P}(X_{t+n} = j | X_t = i), i, j\epsilon S, t, n\epsilon \mathbb{N}_0 \quad (7)$$

Then for each $j\epsilon S$ holds:
$$p_j^{(n)} = \mathbb{P}(X_n = j) = \sum_{i\epsilon S} \mathbb{P}(X_n = j, X_0 = i) = \sum_{i\epsilon S} \mathbb{P}(X_n = j | X_0 = i) \cdot \mathbb{P}(X_0 = i) = \sum_{i\epsilon S} p_i^{(0)} p_{ij}^{(n)} \qquad (8)$$

Thus, the state vector at any step can be determined with a known transition probability matrix and a known initial state vector:

$$p^{(n)} = p^{(0)} \cdot P^{(n)} \qquad (9)$$

For irreducible ergodic Markov chain (Hillier & Lieberman, 2000) it is possible to predict long-run probabilities, i.e. probabilities that chain will be in each state after a large number of transitions. Then:

$$\lim_{n\to\infty} p_{ij}^{(n)} = \pi_j$$
$$(10)$$

where π_j denote steady-state probabilities of Markov chain. For every $j\epsilon S$ following equations hold:

$$\pi_j = \sum_{i\epsilon S} \pi_i p_{ij}, \qquad (11)$$
$$\sum_{j\epsilon S} \pi_j = 1. \qquad (12)$$

4. Sales and number of customers forecasting by using Markov chain method

This section describes the problem of sales and number of customers forecasting in a particular sales company from the real system. For the identified real problem, model construction and forecasting problem solution by using Markov chain method is presented.

4.1. Defining sales forecasting problem

The problem of forecasting is presented for a company whose main activity is the distribution and sale of expensive cosmetic products. Given the main activity, it is of great importance for the company to know the market situation and care for customers. In order to determine how many products must be ordered, a sales plan is made for each quarter. The sales plan for certain products is made in such a way that, based on after-sales activities, registered customers are surveyed (usually by telephone) and information is obtained on their satisfaction with the purchased products. Based on the information received from customers and available information from the previous quarter, the demand for certain products is forecast.

In the continuation of the paper, the sales planning for the period from April 1, 2020 to June 30, 2020, ie for the second quarter in 2020, is presented. It is intended to release a sales plan for the 3 best-selling perfumes in the company: Tom Ford Oud Wood, Byredo Pulp and Creed Aventus For Her. The company has 2,200 registered (loyalty) customers and data on their purchased products in the previous quarter (from October 1, 2019 to December 31, 2020). Out of 2,200 registered customers, 433 bought some of the above three perfumes mentioned in that quarter. 185 customers bought Tom Ford Oud Wood perfume, 116 customers bought Byredo Pulp perfume and the rest of customers bought Creed Aventus For Her. The mentioned customers were contacted and asked about their satisfaction with the purchased products, and whether they will continue to buy that product or will decide on another one (where they were offered the remaining two perfumes). Out of the 185 customers who bought the Tom Ford Oud Wood perfume, 118 of them said they would buy the same perfume next time. The remaining 67 customers said they would not buy the same perfume. When asked if they might buy another perfume (they were offered Byredo Pulp and Creed Avantus For Her), 42 of them said they would like to buy Creed Avantus For Her, and 25 of them said they would like to buy Byredo Pulp. Out of the 116 customers who bought Byredo Pulp perfume, 87 said they could buy the same perfume next time. The remaining 29 respondents answered that they would not buy the same perfume. When asked if they might buy another perfume (they were offered Tom Ford Oud Wood and Creed Avantus For Her), 18 said they would like to buy Tom Ford Oud Wood perfume and the other 11 said they would like to buy Creed Avantus perfume. For Her. Of the 132 customers who bought the perfume Creed Avantus For Her, 101 of them replied that they would buy the same perfume again. The remaining 31 customers replied that they would not buy the same perfume. Asked if

they might buy another perfume (they were offered Tom Ford Oud Wood and Byredo Pulp), 10 said they could buy Tom Ford Oud Wood perfume and 21 said they could buy Byredo Pulp perfume. It is important to note that the data on the number of registered customers and the number of perfumes sold are true. The data obtained from the customer survey are approximate, ie they are obtained in approximate percentages, and authors of this paper converted them into specific numerical values.

4.2. Sales forecasting problem solving

To solve the problem of sales forecasting first it is necessary to build a model that requires determining the movement of changes in consumer behavior when buying a product from different manufacturers. This step will be solved using a spreadsheet based on the data from the problem described above. To simplify the display, Tom Ford Oud Wood is labelled with A, Byredo Pulp with B and Creed Avantus For Her with C. The problem is given in Table 1.

In the second step, the calculation of the transition probability matrix $P = [p_{ij}]$ was performed based on the data from the previous table. The transition probability matrix P is:

$$P = \begin{bmatrix} \frac{118}{185} & \frac{18}{116} & \frac{10}{132} \\ \frac{25}{185} & \frac{87}{116} & \frac{21}{132} \\ \frac{42}{185} & \frac{11}{116} & \frac{101}{132} \end{bmatrix} = \begin{bmatrix} 0.638 & 0.155 & 0.076 \\ 0.135 & 0.75 & 0.159 \\ 0.227 & 0.095 & 0.765 \end{bmatrix}.$$

Interpretation of the matrix P is possible by rows and columns, depending on the transition information which indicates an increase or decrease in sales of one product in favor of another product. For example, the first row of the matrix is interpreted as follows: 63.8% of customers who bought perfume A will continue to buy perfume A. 15.5% of customers who bought perfume B now will buy perfume A. 7.6% of customers who bought perfume C, from now on will buy perfume A. The interpretation for the columns is similar. For example, the

interpretation of second column is: 15.5% of customers who bought perfume B, now will buy perfume A. 75% of customers who bought perfume B will continue to buy perfume B. 9.5% of customers who bought perfume B, now will buy perfume C. The assumption on which Markov chains are based is that the probability of the next event (in our case the choice of perfume by the customer in the 2nd quarter of 2020) depends on the output of the previous event (in our case the choice of perfume by the customer in the 1st quarter of 2020). Therefore, to calculate the expected structure of perfume sales for the 2nd quarter in 2020, it is first necessary to calculate the sales structure in the 1st quarter in 2020, i.e. it is necessary to calculate the elements of the matrix A_0. The structure of sales in the 1st quarter is shown in the Table 2.

After the transition probabilities matrix P and the sales structure in the 1st quarter of 2020, matrix A_0, the share of perfumes in the sales of each of the perfumes in the 2nd quarter of 2020 can be calculated. In order to obtain the share of perfume in sales, the matrix P is multiplied by the matrix A_0, which is also the fourth step in the model building.

$$A_1 = P \cdot A_0 = \begin{bmatrix} 0.638 & 0.155 & 0.076 \\ 0.135 & 0.75 & 0.159 \\ 0.227 & 0.095 & 0.765 \end{bmatrix} \cdot \begin{bmatrix} 0.337 \\ 0.307 \\ 0.356 \end{bmatrix}$$
$$= \begin{bmatrix} 0.290 \\ 0.332 \\ 0.378 \end{bmatrix}$$

In the newly calculated matrix A_1, the first element is smaller than the first element in matrix A_0, so demand for Tom Ford Oud Wood is expected to fall in the 2nd quarter. The second element in matrix A_1 is higher than the second element in matrix A_0, so demand for Byredo Pulp is expected to grow in the 2nd quarter. The same goes for the perfume Creed Avantus For Her. By repeating the fourth step, only if the matrix P was multiplied by the matrix A_1, the share of perfumes in the sale of each of the perfumes in the 3rd quarter of 2020 would be obtained. The fourth step can be repeated for each subsequent period.

The observed company is also interested in a steady-state. Assuming that in the meantime the company does not introduce a new perfume in its sales range or competition does not appear, after a certain number of quarters, a steady-state of participation would be achieved. To determine the steady-state, it is necessary to calculate a matrix A_{m+1} that will satisfy the following matrix assumption:

$$P \cdot A_{m+1} = A_{m+1}$$

By incluing transition probabilities matrix in the previous equation and the following expression is obtained:

$$\begin{bmatrix} 0.638 & 0.155 & 0.076 \\ 0.135 & 0.75 & 0.159 \\ 0.227 & 0.095 & 0.765 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

After solving the system of equations, the matrix of the steady-state was obtained:

$$\begin{bmatrix} 0.241 \\ 0.373 \\ 0.386 \end{bmatrix}$$

The solution to the problem shows the shares of sales: Tom Ford Oud Wood perfume 24.1%, Byredo Pulp perfume 37.3% and Creed Avantus For Her perfume 38.6%.

Table 1. Tabular presentation of sales forecast problem

Product	Sold quantity - 4th quarter 2019.	Falling sales			Quantity - loyal customers	Sales growth			Quantity sold - 1st quarter 2020 (survey)
		A	B	C		A	B	C	
A	185	0	25	42	118	0	18	10	146
B	116	18	0	11	87	25	0	21	133
C	132	10	21	0	101	42	11	0	154

Source: Petek (2020, p. 15)

Table 2. Tabular presentation of the sales structure in the next period

Product	Quantity sold in the 1st quarter of 2020	Sales structure in the 1st quarter - elements of the A_0 matrix
A	146	$\frac{146}{433} = 0.337$
B	133	$\frac{133}{433} = 0.307$
C	154	$\frac{154}{433} = 0.356$
Total	433	1.00

Source: Petek (2020, p. 16)

4.3. Defining customer forecasting problem

At the end of 2019, the observed company expanded its sales by opening a new retail space in an area of about 9000 inhabitants. The market research conducted by the sales department came to the information that in 2019 on the sample of 600 customers, 423 customers remained customers and 177 of them became non-customers. Also, out of 1100 non-customers, 785 remained non-customers and the remaining 315 became customers. As the neighborhood has about 9000 inhabitants, they represent potential buyers at the time of the store's opening, which was in the last quarter of 2019. The company is interested in how many customers it will have in the area in 2020 during the summer season, which is most pronounced in June, July and August, which represents the 3rd quarter of 2020.

4.4. Customer forecasting problem solving

In the representation of the number of customers forecasting problem in matrix form, the notation C_1 denotes the number of customers at time t, and the notation NC_t denotes the number of non-customers at time t. It is assumed that the sales space at time $t = 1$ is estimated to have 9000 potential customers and in that period they were all non-customers. As retail space was opened in the last quarter of 2019, that quarter marks time $t = 1$. The company is interested in

how many customers it will have in the summer season, i.e. in the 3rd quarter of 2020. Therefore, it is necessary to determine the number of customers in the time $t = 4$ ($t = 2$, indicates the 1st quarter in 2020, $t = 3$ indicates the 2nd quarter in 2020 and $t = 4$ indicates the required, 3rd quarter in 2020). The following describes the problem described in matrix form

$$
\begin{array}{cc}
\text{Time } t = 1 & NC_1 \quad C_1 \\
\text{Time } t = 0 & \\
NC_0 & \begin{bmatrix} 785 & 315 \\ 177 & 423 \end{bmatrix} \\
C_0 &
\end{array}
$$

For the purposes of the forecast, the transition probability matrix was also calculated:

$$
\begin{array}{cc}
\text{Time } t = 1 & \\
\text{Time } t = 0 & NC_1 \qquad C_1 \\
NC_0 & \begin{bmatrix} 0.714 & 0.286 \\ 0.295 & 0.705 \end{bmatrix} \\
C_0 &
\end{array}
$$

After the calculations were performed, by using the previously calculated transition probability matrix, a forecast of the number of customers for the third quarter was obtained. It can be concluded that the company in the newly opened retail space during the summer season in 2020 can expect a total of 4104 customers. The presented solutions to the problem can be a good basis for the company to make

decisions about the quantity goods orders for the period for which the forecast was made.

5. Software application based on the Markov chain method

This section describes software application development for predictive problems solving by using Markov chains and a presentation of the operation of the application on the forecasting problem presented in the previous sections. The representation of system entities and their behavior are documented by using the standardized Unified Modeling Language (UML). Two types of class diagram diagrams and activity diagrams were used.

5.1. Class diagram for sales forecasting and customer forecasting functionality

The class diagram shows the system static structure that is always valid at every moment of the system life cycle. This diagram defines the elements of classifiers (concrete classes, abstract classes, interfaces, packages…) and the relationships between them. The internal structure of an entity, in this case a class, is represented by attributes and operations or methods. For the needs of this application, a total of 10 classes have been created, of which 5 classes are forms, i.e. user interfaces, and the rest are entity classes.

Figure 1. Class diagram
Source: Petek (2020, p. 23)

In the diagram, a black rhombus indicates that the class on whose side it is a whole, and the other class that it is part of that whole. In other words, the class that forms part of that whole cannot exist without the class that is the whole. A white rhombus indicates that the class on whose side it is located uses objects or methods of another class (Figure 1).

5.2. Activity diagram for sales and number of customers forecasting functionality

An activity diagram is a behavior diagram in which all states are either actions or sub-actions, with each action beginning at the end of the previous action. An activity diagram was used to describe the activities that occur in an operation, to describe interactions, and to describe how they are used. The specificity of the developed

application is that it consists of two functionalities: sales forecasting and forecasting the number of customers.

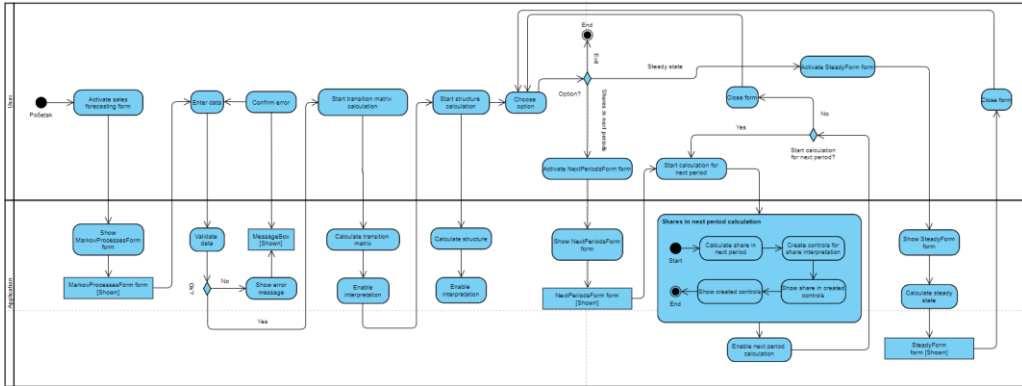Therefore, these two functionalities are shown separately in Figure 2 and Figure 3.

Figure 2. Activity diagram for functionality - sales forecasting
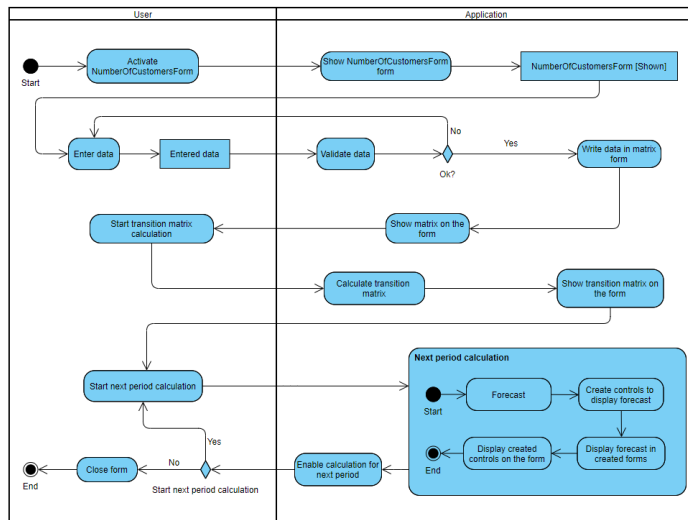Source: Petek (2020, p. 24)

Figure 3. Activity diagram for functionality - number of customers forecasting
Source: Petek (2020, p. 25)

5.3. Demonstration of the sales forecasting application functionality

The sales forecasting functionality starts after the user activates the sales forecast form (interface) in the main menu (Figure 4).
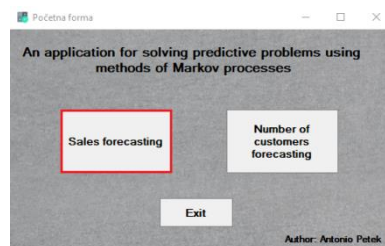
Figure 4. Main menu of the application - sales forecasting
Source: Petek (2020, p. 26)

The application displays the activated form and the user enters data. For each data entered, the application validates data. If the data is entered incorrectly, the application prints an error message and the user must re-enter the incorrect data. After the data is entered correctly, the user starts the calculation of the transition probability matrix. The application in the background does the calculation and allows the interpretation of the results. After that, the user starts the calculation of the structure and the application in the background does the calculation and allows the interpretation of the results (Figure 5). The user then has three options: output for the end of functionality, calculation of the share in the following periods and calculation of the steady state. If he chooses the second option, the application opens a form for calculating the share in the following periods in which the user starts the calculation. The application displays the calculation on the form and allows the user to repeat the procedure. If the user decides to repeat, the previously described procedure is repeated, and if not, the user closes the form.

Figure 5. Sales forecast form
Source: Petek (2020, p. 34)

Based on the presented data (Figure 5), the identity of the forecasting problems results obtained in the previous chapter can be observed, which confirms the correctness of the developed application.

With the software application, the user has the ability to calculate the steady-state. In this case, the application opens the form for calculating the share of steady-state (Figure 6). After that, the user closes the form and that is the end of the functionality. As in the previous figure, the identity of the solution is confirmed by using the software application.

Figure 6. Steady-state calculation form
Source: Petek (2020, p. 36)

5.4. Demonstration of the number of customers forecasting application functionality

The number of customers forecasting functionality starts after the user activates the number of customers forecasting form (interface) in the main menu (Figure 7).
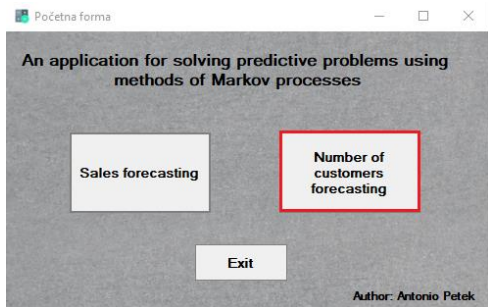

Figure 7. Main menu of the application - number of customers forecasting
Source: Petek (2020, p. 36)

The application displays the activated form and the user enters data. As in the previous functionality, the application validates data after entering. If they are incorrect, it warns the user. If the data is correct, the application writes data in matrix form and displays the matrix on the form. After that, the user starts the calculation of the transition probability matrix and the application performs the calculation.

The application displays the obtained data on the form and allows the user to run the calculation of the forecast for the next period. The application then calculates the forecast in the background and gives the user an overview of the form and allows the calculation for the first following period. The user can start the calculation of the forecast for the next period and the previously described procedure is repeated (Figure 8). Another option provided by the application is that the user does not decide to start the forecast calculation and closes the form. Then, that is the end of the functionality.
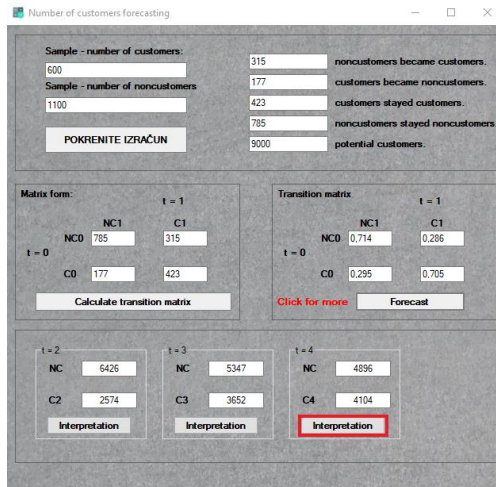

Figure 8. Form for the number of customers forecasting – calculation performed
Source: Petek (2020, p. 41)

The final solution (Figure 8) indicates that in the third quarter the number of customers intending to make a purchase in the newly opened sales point of the observed company will be 4104, which also corresponds to the calculation obtained in Subsection 4.4. This also shows the correctness the software solution for the number of customers forecasting.

6. Conclusion

Today's business world requires quick and quality decisions with a detailed analysis of all factors that affect them. In achieving the set goals, companies have at their disposal various mathematical methods developed for predicting future phenomena. One such method is the Markov chain method, which is widely used due to the accuracy of the prediction results.

This paper describes two specific problems from the real system related to sale of cosmetic products forecasting and the number of customers forecasting. Namely, when preparing a sales plan, the approximate number of products that could be sold in the analyzed period is assumed. By including in the Markov chain model sales data from the

previous period and data obtained from market research, values that represent the approximate amount of products that will be sold in the next period are obtained. According to these results, procurement is planned in order to avoid the accumulation of products in the warehouse, and thus achieve savings. A software application for the described problem of forecasting has been developed. The application is documented by using two types of UML diagrams: class diagram and activity diagram. The class diagram describes the entities that are in the system, their internal structure and connections with other entities, while the activity diagram describes how to use the application. Through the presentation of the functionality of the developed software application, it was shown that the results obtained by the software application coincide with the results obtained by the calculation procedure, based on which it can

be concluded that the application gives valid and reliable results.

This paper presents the functionalities of the developed software application on the problem related to the distribution and sale of cosmetic products. The same functionalities of the application are intended to be tested in future research on other branches of production and service activities with the aim of generalizing and expanding the use of the application. For future research it is expected to generalize the use of the developed software application with minimal modifications. This is supported by the fact that Markov chains have been used in many other areas, such as informatics, physics, chemistry, biology, finance, and because of the ease use and reliable results on which adequate business decisions can be made to achieve better business results.

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