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INTEGRATING EXPERT JUDGMENTS INTO THE PROCESS OF MEASURING THE EFFICIENCY OF THE NATIONAL ECONOMY: DEA-AHP APPROACH

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Abstract: *The paper measures the efficiency of the national economy of Republic of Serbia, in the period 2007-2017 year, marked by the global financial crisis and natural disasters that caused catastrophic consequences. First, the DEA method was applied, and then expert assessments were integrated into the model using the AHP method, thus overcoming the problem of irrelevance of individual variables. The results showed that the introduction of additional restrictions, based on expert judgments, for the weighting coefficients of the input variables affected the reduction of technical efficiency in the observed period compared to the baseline model, but that in both cases in 2007 the national economy was technically efficient. Robustness analysis of the obtained efficiency values was performed using bootstrap DEA, thus confirming the consistency of the results.*

Keywords: *Data envelopment analysis; Analytical hierarchical process; National economy; Gross domestic product; Efficiency; Weights restrictions; Expert judgments.*

1. Introduction

Macroeconomic stability is the primary goal of every country's economic policy. The primary goal of macroeconomic stabilization policy should be to achieve stable economic growth. Price stability plays an important role through the impact on investment decisions. Since unemployment is a major cause of poverty, the key task is to keep the economy at a high level of employment. To examine macroeconomic stability, macroeconomic performance is most often monitored, including gross domestic product (GDP) growth, inflation rate, and employment levels. High growth rates, as indicated by changes in gross domestic product, GDP, low inflation rates as shown by changes in consumer price indices (CPIs) and high employment rates are the main targets or missions of national macroeconomic policy. The Gross Domestic Product is the main component used to quantify the results of obtained economic growth and validates government action and the efficiency of state governance (Bodislav, 2014. p.24)

As from the economic point of view, every economy is efficient if it increases its GDP through the reduction of used resources, through technological, behavioral and economic changes, the paper deals with the technical efficiency of the national economy of the Republic of Serbia (RS). For this purpose, the DEA method was used to measure technical efficiency, which enables the identification of the most successful decision units (DMUs), of which a linear combination defines the efficiency limit. In relation to this limit, the technical efficiency of the national economy is determined for the

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period 2007-2017, through the use of basic resources such as energy, labor and capital to maximize GDP production. In addition, in order not to ignore the relative importance of these GDP determinants, the starting model has been expanded with additional constraints, which define the lower and upper limits of the weighting coefficients of input variables, which were achieved by applying the Analytic Hierarchy Process method.

2. Literature review

Economic efficiency is a broad term typically used in microeconomics in order to denote the state of best possible operation of a product or service market (Petrou, 2014). Although in the literature the notion of national economy as a unit of decision in terms of DEA methods almost does not exist, there are several references in the scientific and professional literature that point to the problem of measuring the efficiency of the national economy. Economists have long been interested in theoretical and empirical analysis of technical efficiency in mathematical programming, especially the DEA framework, with different combinations of macroeconomic indicators taken as input and / or output variables in its application. Debreu (1951) offers two main reasons why inefficiencies can be observed: (i) market failure, and (ii) unprofitable behavior resulting from institutional structures other than private property and individual property rights. Although many applications of DEA-derived efficiency concepts were at the level of individual "production" and sub-national units (firm, school, region, etc.), the resource utilization ratio proposed in Debreu (1951) (and equivalent to input-based efficiency results), designed is to measure the deviation of economic systems from the characteristics of general equilibrium. Lovell (1990) applied the technique of mathematical programming of performance measurement to construct "best practice" and established a scalar measure of the macroeconomic performance of an economy. However, there are no input variables in this application and it contains only four results (GDP growth, employment, trade balance and price stability). Moreover, the study was conducted on a small scale of 10 economies in Asia, with a focus on Taiwan, for different times, so it was difficult to come up with a convincing overall performance ranking. Also, Lovell et al. (1995) study the macroeconomic performance of 19 OECD countries in the period 1970-1990 and develop an alternative DEA model, which incorporates service gaps into the performance evaluation process in an economically meaningful way. Martić et al. (2001) used the DEA method to assess how well regions in Serbia use their resources. Based on input and output data, they applied an output-oriented DEA model, and 17 of the 30 regions appear to be efficient. By the way, Despotis (2005) extended the applicability of the DEA model with variable returns to the scale to assess the relative efficiency of countries in Asia and the Pacific in converting income into human development. However, balanced economic growth must be accompanied by the conservation of resources and the environment in a sustainable world. High growth rate (as shown by the change in gross domestic product), low inflation rate, low unemployment rate and favorable trade balance are the four main goals or objectives of macroeconomic creators of national macroeconomic policy. These performance indices are referred to in the literature as the "magic diamond" of the OECD (Organization for Economic Co-operation and Development) with four foundations synonymous with the four indicators. The sum of inflation and the unemployment rate define Okun's undesirable "poverty index" and provide a pessimistic measure of a nation's macroeconomic performance. An alternative undesirable measure is provided by the Calmfor index, defined as the difference between the unemployment rate and the normalized trade balance. Cherchye (2001) used a model based on the DEA method to evaluate different economic goals. The study provided a comparison of several synthetic indicators that combined four separate indicators into one statistic. Milner & Weiman-Jones (2003), investigate the impact of the size of a country in terms of its area on overall national efficiency by applying the nonparametric programming method to a group of 85 developing countries, in the period 1980-1989. The results showed that there is a strong positive impact. The results showed that there is a strong positive relationship between development and efficiency and evidence of a positive impact of trade policy openness on overall efficiency. Chien and Hu (2007) analyzes the effects of renewable energy on the technical efficiency of 45 economies during the 2001–2002 period through data envelopment analysis (DEA). They show that Increasing the use of renewable energy improves an economy's technical efficiency. Menegaki (2013) uses data envelopment analysis (DEA) for the purpose of calculating inefficiencies in the European countries' growth using as main inputs, energy consumption, carbon emissions, employment and capital but also with a particular focus on renewable energy sources (RES) consumption. Results show that countries with remarkable renewable energy performance have medium to low efficiency, while renewable energy laggards are among the most technically efficient countries in Europe. There is also some evidence of limiting country size to efficiency when other impacts are abstracted. Suzuki et al. (2016) measure the economic efficiency of energy and the environment with two inputs (energy consumption and population) and two results (outputs) (CO_2 emissions and GDP), including a fixed input factor related to the population. Wang and Lee (2018) measure and predict macroeconomic performance using DEA methods on the example of developed European countries and Asian developing countries, in the period 2013-2016 and 2017-2020. Using four macroeconomic indicators, government gross debt, GDP growth rate, inflation rate and unemployment rate, they construct a scalar measure of macroeconomic performance and show that Switzerland, Singapore and the US are the most economically successful countries. Vaz and Pereira (2019) propose a framework for the application of the DEA method for assessing the technical efficiency of 26 European countries in the last five years, within the current energy policy in 2020. DEA is used to assess efficiency supplemented by bootstrapping to obtain statistical conclusions. It has been observed that the efficiency of economies has increased by about 13% on average, since 2009. The results obtained indicate that the energy policy efforts developed in each country do not jeopardize the improvement of their efficiency. Also, several studies on measuring and assessing the macroeconomic and developmental performance of regions, cities, provinces, and nations have been conducted and published in the literature (Mohammad, 2007).

3. Methodology

Data envelopment analysis (DEA) is a mathematical, nonparametric approach for calculating efficiency, which does not require a specific functional form. It is used to evaluate the performance of Decision Making Units (DMUs) by reducing multiple input variables to a single virtual input and multiple output variables to a single virtual output using weight coefficients. The DEA methodology has proven to be adequate especially when assessing the efficiency of non-profit organizations operating outside the market, because in their case performance indicators such as income and profit do not measure efficiency satisfactorily. Unlike typical statistical methods, data envelope analysis is based on benchmarking, comparing each decision unit with only the best DMU. All data on input and output variables for each of the n decision units are inserted into a particular linear program which is actually one corresponding to the n formed DEA models. Thus, the efficiency of the observed decision-making units is evaluated, which in fact represents the ratio of the weighted sum of the output variables and the weighted sum of the input variables. Data envelope analysis is about relative efficiency because decision-making units are observed in relation to others.

The ratio DEA model, also known as the CCR model (Charnes et al., 1978) measures the efficiency of the DMU_j as the maximum value of the quotient of the weighted sum of outputs and the weighted sum of inputs, i.e.:

$$(\max)h_k = \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \quad (1)$$

s.r.

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i\, x_{ij}} \leq 1, \text{ j=1,2,..,}n$$

$$u_r \geq 0, \text{ r=1,2,..,}s$$

$$v_i \geq 0, \text{ i=1,2,..,}m$$

Where they are:

h_k - relative efficiency of the DMU;

n - number of DMU to be compared;

m - number of input variables;

s - number of output variables;

u_r - weight coefficient for output r;

v_i - weighting factor for input and.

The CCR ratio model calculates the overall technical (radial) efficiency, which includes both pure technical efficiency and efficiency as a consequence of different business volumes. The value of the objective function h_k ranges between 0 and 1. If the value of h_k is equal to 1, the k-th DMU is relatively efficient, and if it is less than 1, the DMU_k is relatively inefficient and the value of h_k shows the required percentage reduction of input to become effective. The above model of fractional linear programming has two operational forms, depending on the orientation. The first form maximizes the virtual sum of the outputs of the j-th DMU, where its virtual input is 1 and is known as the input-oriented model, while the second, used in this paper, minimizes the total virtual input, where the virtual output is 1, and is known is like an output-oriented model. Input-oriented efficiency scores range between 0 and 1, while output-oriented efficiency scores range from 1 to infinity, with in both cases DMU_j whose efficiencies are equal to 1 being relatively efficient. Depending on the purpose of the analysis, the orientation of the model is chosen, so the analyst should articulate the purpose of the analysis, input reduction, output extension, or both, bearing in mind that from the DEA method point of view, regardless of the choice of orientation, effective or best practice (Cook et al, 2014). Important assumptions on which the valid application of the DEA model is based are defined by the principle of homogeneity, i.e. similarity of decision units, property of positivity of input and output variables, property of isotonicity which implies that increase of some input results in the same increase of output without decrease of any other input, as well as optimal number of input and output variables that fully measure the effect of decision units. all decision-making units (for more details on the practical application of the DEA method, see: Dyson et al. 2001; Sarkis, 2002; Sherman & Zhu, 2006; Cooper et al, 2007; Cook et al, 2014; etc.).

4. Problem description and structuring

4.1 Output oriented DEA CCR model for measuring the technical efficiency of the RS national economy

Starting from the goal of the research, it is clear that the interest is to optimize the achieved results as goals of the economic policy of the Republic of Serbia. And as, in addition to certain theoretical controversies, gross domestic product is a sublimated expression of the achieved macroeconomic results, for the purposes of analyzing the efficiency of the RS national economy, an output-oriented CCR ratio model (CCR - O) was chosen, whose appropriate multiplicative form, in general, is:

$$(\min)\theta_k = \sum_{i=1}^{m} v_i\, x_{ik} \quad (2)$$

$$\sum_{r=1}^{s} u_r\, y_{rk} = 1$$

$$\sum_{i=1}^{m} v_i\, x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0, \text{ j=1,2,…,n}$$

$$u_r \geq 0, \qquad r = 1,2,,s$$

$$v_i \geq 0, \qquad i = 1,2,..,m$$

While its dual form, which is more often resolved, is:

$$(\max)\, \emptyset_k + \varepsilon\left(\sum_{i=1}^{m} s_i^{-} + \sum_{r=1}^{s} s_r^{+}\right) \quad (3)$$

$$\sum_{j=1}^{m} x_{ij}\, \lambda_j + s_i^{-} = x_{ik} \qquad i = 1,2,..,m$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^{+} = \emptyset y_{rk} \quad r = 1,2,..,s$$

$$\lambda_j,\ s_r^{+},\ s_i^{-} \geq 0 \qquad j = 1,2,..,n;\ i = 1,2,..,m;\ r = 1,2,..,s$$

Where s_i^{-} i s_r^{+} are dual variables that speak of the necessary individual reduction of the *i-th* input and increase of the *r-th* output of the *k-th* DMU_s in order to become efficient. Since they represent a complement to equality in the system of constraints, they are called complementary variables. The dual variable λ_j represents the dual weight that shows the importance assigned to DMU_j (j = 1, 2,.., n) in defining the input-output mix of the hypothetical composite unit with which DMU_k will be directly compared. DMU_k is technically efficient, if and only if, for the optimal solution ($\lambda^*, s^{+*}, s^{-*}, \emptyset_k^{\,*}$) the conditions are met: $\emptyset_k^{\,*} = 1$; $s^{+*} = 0$; $s^{-*} = 0$. In an output-oriented model, an inefficient unit becomes efficient through a proportional increase in its results, while the proportions of the input remain unchanged. In the output-oriented model, the relative efficiencies are equal to or greater than 1, with those DMUs whose efficiency is 1 relatively effective and those whose efficiency is greater than 1 inefficient. However, as shown by Cooper et al, (2007), the efficiency of the input oriented model can be obtained as a reciprocal of the efficiency value of the output oriented model. A DMU is said to be Pareto-efficient if it is not possible to raise any of its output levels without lowering at least one of its output levels and / or without increasing at least one of its input levels (Cooper et al, 2007). Using the optimal solution ($\lambda^*, s^{+*}, s^{-*}, \emptyset_k^{\,*}$) the target values for the DMU to be decided can be determined, where:

$$x_i^{\,*} = \emptyset_k^{\,*} x_i - s_i^{-} = \sum_{j=1}^{n} x_{ij}\lambda_j^{\,*}, \qquad (4)$$
$$i = 1,2,..,m$$

and

$$y_r^{\,*} = y_r + s_r^{+} = \sum_{j=1}^{n} y_{rj}\, \lambda_j^{\,*}, \quad (5)$$
$$r = 1,\ldots, s$$

As numerous studies suggest, the number of DMUs in the observed set should be sufficiently higher than the total number of inputs and outputs, because there is a danger that most DMUs will be classified as efficient precisely because of the DEA's tendency to present each unit in the best possible light. (Charnes et al, 1978; Dyson, 2001; Sarkis, 2002; Cook et al, 2014). However, such a rule is not essential, nor is it statistically grounded, but is often followed for purely practical reasons, so as not to reduce the discriminatory power of the model, so it does not necessarily have to be met. Other rules on the optimal number of input and output variables can be found in the literature, such as the general rule $m + s < n / 3$ or $m \times s < n$ and $m + s < n / 2$ (Cooper, Seiford, & Tone, 2007). In this sense, different approaches to the choice of inputs and outputs are known in the scientific and professional literature, most often correlation and regression analysis but also other ways to solve the problem of a large number of input parameters (multivariate statistical analysis, Jerkins & Anderson, 2003; maximization of correlation between DEA efficiency index and external performance index, Edirisinghe & Zhang, 2007; two-phase heuristic algorithm, Morita & Avkiran, 2009, etc.)

The choice of variables in this case was made based on the analysis of papers and publications that investigate the problem of efficiency of national economies (Vaz & Ferreira, 2019; Milner & Weyman-Jones, 2003; Chien & Hu, 2015; Milenković et al, 2017; OECD 2001; Lábaj et al, 2013; etc.), and refers to labor, capital and energy as input variables, i.e. used resources and GDP as an output variable. The most commonly used classification of inputs in measuring business efficiency and productivity includes five categories: capital (C), labor (L), energy (E), material input (M), and purchased services (S). The collection and use of data related to these categories in measuring productivity is sometimes called the KLEMS approach, with the latter three categories viewed as a single input (Coelli et al, 2005, p. 141). In companies that produce a single product, the output is often taken as annual output, or its value, as well as the realized operating income (Coelli et al, 2005, p. 136)

So the assumptions on which the DEA CCR-O model is based are:

1) The observed time period is 2007-2017; at the same time, they form a set of observed DMUs;

2) Input variables are: I_1 - labor force participation rate (% of total population); I_2 - net capital accumulation (% of GDP); I_3 - Energy consumption per capita (in kg of oil equivalent);

3) The output variable is O_1 - GDP growth rate (%), as the most commonly used measure of total economic activity of a country;

The need to choose an output-oriented model becomes more obvious here, bearing in mind that the input variables I_1 and I_2 can hardly be changed in the short run, while in terms of energy consumption the situation is somewhat more favorable. The applied model can be modified in different ways, depending on the goal. Thus Labaj et al, 2013, provide an overview of DEA models for measuring the efficiency of the national economy (Table 1), which differ in the output variables, which in most models include the Gini coefficient as a measure of income inequality or wealth distribution, while the input variables are more or less the same as the model orientation.

Data for selected input and output indicators were collected from *Worldbank database* and corrected in the part related to GDP growth rate, since the application of the DEA model implies that all parameters are positive, so that, in this context, GDP growth rates by years increased by +4 (Table 2), while the correctness of the choice of input and output variables was confirmed by correlation analysis. (Table 3)

Table 1. Overview of output-oriented DEA models for technical efficiency assessment

Model	Description	Inputs	Outputs
Model I (O)	Technical efficiency	Capital; Labor	GDP
Model A (O)	Eco-efficiency	Capital; Labor	GDP; CO_2 emissions;
Model B (O)	Income inequality-adjusted efficiency	Capital; Labor	GDP; Gini;
Model B (O)	Social welfare efficiency	Capital; Labor; CO_2 emissions	GDP; Gini;
Model II	Ecological efficiency	Capital; Labor; CO_2 emissions	GDP
Model III	Income inequality-adjusted efficiency	Capital; Labor	GDP; Gini;
Model I (O)	Social welfare efficiency	Capital; Labor	Results of Models I and II

Source: Labaj et al, 2013.

Table 2. Values of input and output variables of the DEA model for measuring the technical efficiency of the RS national economy

Year	labor force participation rate (% of total population)	net capital accumulation (% of GDP)	Energy consumption per capita (in kg of oil equivalent)	GDP growth rate (%)
2007	51.0	29.1	2248.16	9.9
2008	51.4	30.3	2290.00	9.4
2009	48.9	19.4	2070.23	0.9
2010	50.9	18.5	2141.06	4.6
2011	50.9	20.1	2237.46	5.4
2012	51.3	21.0	2020.40	3.0
2013	52.1	17.6	2080.97	6.6
2014	52.4	17.5	1859.43	2.2
2015	52.0	18.9	1967.38	4.8
2016	53.9	19.1	1986.47	6.8
2017	53.5	21.0	1954.52	5.9

Source: *Worldbank database*

Table 3. The means, standard deviations and correlations

Variable	***Mean***	***S.D***	***1***	***2***	***3***	***4***
1. Labor (I_1)	51.6636	1.3662	-	0.539	0.868	0.846
2 Capital (I_2)	21.1364	4.3971		-	0.771	0.882
3. Energy (I_3)	2077.8255	137.9579			-	0.915
4. GDP (O_1)	1.4091	2.7776				-

$p < 0.05$

4.2 Results of the DEA model and discussion

The technical efficiency for each DMUs (year) was assessed according to a dual model (2), estimating the capacity of each economy in maximizing GDP, taking into account the fundamental contributions (energy, labor and capital). The lowest achieved relative technical efficiency was in 2009 (0.128), and the highest in 2007 and 2013 (Table 4). The relative efficiency of the country in a given year is assessed by comparing it with the best practices observed during the analyzed period, ranging from 2007 to 2017. The analysis of the obtained results shows that the reference set of DMUs consists of 2007 and 2013. The technical efficiency of the national economy of RS in those years is equal to 1, the utilization of available resources was 100%, all dual variables s_i^- i s_r^+ are equal to 0, so the target values of input and output variables are equal to achieved (table 6). Other DMUs are technically inefficient. So, for example, the optimal solution for DMU2009 is:

$$\emptyset^* = 0{,}128;\ \lambda_{2007}{}^* = 0{,}243;\ \lambda_{2013}{}^* = 0{,}701;$$

$$s_1{}^- = 0;\ s_2{}^- = 0;\ s_3{}^- = 65{,}84;\ s_1{}^+ = 6{,}129;$$

Since $\lambda_{2007}{}^* > 0 \ \ i\ \lambda_{2013}{}^* > 0,$ the reference set for DMU_{2009} is R_{2009} {2007, 2013,}. Through these reference values λ^* it is possible to calculate the target values of the input variable I_3 and the output variable O_1, for which the national economy of RS would be technically efficient in 2009, while for the input variables I_1 and I_2 the target values are identical (Table 6). That is, for input variable I_3, it follows:

$I_{3_{2009}}{}^* = \lambda_{2007}{}^* \times I_{3_{2007}} + \lambda_{2013}{}^* \times I_{3_{2013}}$ i.e.,

$I_3{}^* = 0{,}243 \times 2248{,}16 + 0{,}701 \times 2080{,}97 = 2005{,}06,$

Which are approximately the values in the table of realized and target values of input and output variables of the model.

That is:

$$I_{3_{2009}}{}^{*} = \emptyset_k{}^{*} \times I_{3_{2009}} - s_1{}^{-} = 1 \times 2070{,}23 - 65{,}84 = 2004{,}39 \approx 2005$$

Which means that in 2009, the national economy would be technically efficient, it was necessary for energy consumption per capita, expressed in kg of oil equivalent, to be lower by 65.84 (3.18%).

While for the output variable GDP, the target value in 2009 is:

$$O_{1_{2009}}{}^{*} = \lambda_{2007}{}^{*} \times O_{1_{2007}} + \lambda_{2013}{}^{*} \times O_{1_{2013}},$$

$$O_{1_{2009}}{}^{*} = 0{,}243 \times 9{,}9 + 0{,}701 \times 6{,}6 = 7,\ 0323$$

A similar analysis can be done for other DMUs.

The optimal solution of the multiplicative problem gives optimal values of weighting coefficients that express the relative importance of input and output variables, which are for table DMUs, shown in Table 5. These weighting coefficients have managerial and analytical value. For 2009, for example, the optimal solution to the multiplicative problem is:

$$v_1{}^{*} = 0{,}032012;\ v_2{}^{*} = 0{,}321903;\ v_3{}^{*} = 0;$$

$$u_1{}^{*} = 1{,}111111;$$

The value of the weighting factor $v_1{}^{*} = 0{,}032012$ means that the relative technical efficiency of the national economy of RS in 2009 would be higher by 3, 2012% if the labor force participation rate, expressed as a percentage of available number of workers in relation to the total population, was lower by 1%. Analogously, the optimal value of the weighting coefficient of the output variable O1 - GDP growth rate, in the same year is $u_1{}^{*} = 1{,}111111$, which means that the relative technical efficiency would be higher by 111.11%, if the GDP growth rate increased by 1%. The obtained optimal solution satisfies the condition,

$$\sum_{r=1}^{s} u_r\, y_{rk} = 1, \qquad (6)$$

That is, for 2009, it follows:

$$u_{1_{2009}}{}^{*} \times y_{1_{2009}} = 1{,}111111 \times 0{,}9 = 0{,}9999 \approx 1$$

A similar interpretation can be given for other weighting factors as well as for other DMUs. he value of some weighting coefficients in some years is equal to 0 for the optimal solution, which does not reflect the real relative importance of the corresponding input and output variables, because one might think that, say, in 2009, energy consumption as an input variable was not relevant at all, views of the national economy as observed DMUs in that year, given that for its optimal solution $v_3{}^{*} = 0$. Implicitly, in order not to ignore the influence of individual variables, in such situations the model can be extended by additional constraints that define the interval in which relative or absolute values of weight coefficients can move so that their lower limit is greater than zero (Cooper et al, 2007). Weight restrictions usually represent value judgments incorporated in the form of additional constraints on the input and output weights in the multiplier model. These constraints reduce the flexibility of weights and typically improve the discrimination of the DEA model (Cook & Zhu, 2008; Joro & Korhonen, 2015; Thanassoulis et al., 2008, etc.). The use of weight restrictions generally changes the interpretation of efficiency in both the envelopment and multiplier models (Podinovski, 2016). Specifically, in the case of assessing the technical efficiency of the RS national economy, the model could *a priori* incorporate assessments of relative importance on the basis of which the limits within which the values of weight coefficients can move are defined, so that the solution is optimal. As a consequence of the introduction of additional restrictions for weight coefficients, i.e. restrictions by which the value assessment of inputs and outputs is performed, it can lead to a narrowing or widening of the efficiency limit, more often the former. To overcome the problem of ignoring the influence of individual input and / or output variables in this case, the DEA CCR model with the so-called by forming a security region - I type. The term "type I Assurance Regions" was proposed in the paper (Thompson et al, 1986), where the following weight limitations were applied:

$$k_i v_i \leq k_{i+1} v_{i+1} \leq v_{i+2} ,$$

That is:

$$\alpha \leq \frac{v_i}{v_{i+1}} \leq \beta$$

The constraints shown refer to the weights for the input factors. Analogous to them, limitations for the weights of the output factors can be formulated. When the constraints given by the relations given by the relations are applied to the weighting coefficients, the DEA model will always have an admissible solution and there will be at least one efficient DMU. In practical applications, expert opinions were mainly used for their assignment (Podinovski 2016, while in this paper it was done by applying the methodology of analytical hierarchical process (AHP, Saaty, 1980), which defines the intervals in which the relative weight ratios of GDP - labor, capital and Energy.

Table 4. Technical efficiencies of the national economy of RS by years

Year	Score	Rank		Reference (Lambda)		
2007	1	1	2007	1		
2008	0.9421	4	2007	1.008		
2009	0.128	11	2007	0.243	2013	0.701
2010	0.6739	8	2007	0.11	2013	0.869
2011	0.7408	6	2007	0.245	2013	0.737
2012	0.4009	9	2007	0.388	2013	0.552
2013	1	1	2013	1		
2014	0.3446	10	2007	0.176	2013	0.704
2015	0.6997	7	2007	0.224	2013	0.703
2016	0.981	3	2007	0.228	2013	0.708
2017	0.7945	5	2007	0.443	2013	0.46

Table 5. Optimal values of weight coefficients of input and output variables

Year	Score	Rank	v(1)	v(2)	v(3)	u(1)
2007	1	1	0.00291	0.029264	0	0.10101
2008	0.9421	4	0.020651	0	0	0.106383
2009	0.128	11	0.032012	0.321903	0	1.111111
2010	0.6739	8	0.006263	0.062981	0	0.217391
2011	0.7408	6	0.005335	0.05365	0	0.185185
2012	0.4009	9	0	0.091538	0.000283	0.333333
2013	1	1	0	0.056818	0	0.151515
2014	0.3446	10	0	0.124824	0.000386	0.454545
2015	0.6997	7	0	0.057211	0.000177	0.208333
2016	0.981	3	0	0.040384	0.000125	0.147059
2017	0.7945	5	0	0.046545	0.000144	0.169492

Table 6. Achieved and target values of input and output variables

Year	Score	(I1) Data	(I1) Projection	(I2) Data	(I2) Projection	(I3) Data	(I3) Projection	(O1) Data	(O1) Projection
2007	1	51	51	29.1	29.1	2248.16	2248.16	9.9	9.9
2008	0.94	51.4	51.4	30.3	29.3282	2290	2265.79	9.4	9.97765
2009	0.128	48.9	48.9	19.4	19.4	2070.23	2004.39	0.9	7.02929
2010	0.67	50.9	50.9	18.5	18.5	2141.06	2056.26	4.6	6.82617
2011	0.741	50.9	50.9	20.1	20.1	2237.46	2084.71	5.4	7.28971
2012	0.40	51.3	48.53	21	21	2020.4	2020.4	3	7.48225
2013	1	52.1	52.1	17.6	17.6	2080.97	2080.97	6.6	6.6
2014	0.34	52.4	45.62	17.5	17.5	1859.43	1859.43	2.2	6.38444
2015	0.699	52	48.07	18.9	18.9	1967.38	1967.38	4.8	6.86055
2016	0.98	53.9	48.53	19.1	19.1	1986.47	1986.47	6.8	6.93169
2017	0.79	53.5	46.59	21	21	1954.52	1954.52	5.9	7.42633

4.3 CCR AR - I DEA model for assessing the relative technical efficiency of the RS national economy

To assess the relative importance of GDP variables, a simple model of Analytic Hierarchy Process (AHP; Saaty, 1980) with two levels was formed. At the first level is the goal of the model, while at the second level are the variables whose relative importance we determine in relation to the goal (Figure 1). The question that arises by comparing the identified variables is: which variable do we consider to have a greater impact on GDP, and if so,

by how much on the Saaty scale 1-9 (Appendix, *Tables 1 and 2*)? Comparisons and assessments in this case were made jointly by the

authors (2) and fellow university professors whose narrow scientific field is macroeconomics and economic development (2). To calculate the final weights of factors observed and determine group preferences, and to form group ranking of alternatives, geometric mean is used, as a way to combine and objectify rankings in cases where there are multiple decision-makers (Saaty and Peniwati, 2008):

$$w_i = \sqrt[K]{\prod_{k=1}^{k=K} \times w_{ik}} \quad \forall \ i$$

where w_i is the final weight of *i*-th factor, and w_{ik} the relative weight of *i*-th element, calculated on the basis of *k*-th evaluator.

Thus, geometric mean of relative importance of observed determinants of GDP, derived from the same number of respondents' decision-making matrices (2), in this case will be:

$$w_i = \sqrt[4]{\prod_{k=1}^{k=4} w_{ik}} \quad \forall \ i$$

Thus, for factor *Capital*, the final priority derived from 2 rankings will be:

$$w_C = \sqrt[4]{w_{C1} \times w_{C2} \times w_{C3} \times w_{C4}} = 0.24985$$

The relative importance of the remaining factors is calculated in an identical way. After the comparisons, which have n (n-1) / 2, i.e. in this case three, for each decision maker, and calculations of the geometric mean, the results are shown in Table 7. The greatest relative importance for GDP, in this sense, is energy consumption (0.6548), followed by capital (0.24985) and finally labor as a variable (0.09533).

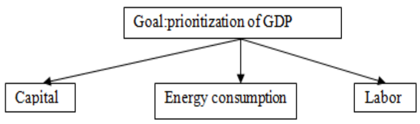

Figure 1. AHP model for assessing the relative importance of GDP components

Table 7. Relative importance of GDP components

Name	Normalized	Ideal
Capital	0.24985	0.38157
Energy consumption	0.65480	1.0
Labor	0.09533	0.14559

Inconsistency: 0.01759

Where the values of the quotient are:

$$\frac{w_{I_1}}{w_{I_2}} = 0{,}382$$

$$\frac{w_{I_1}}{w_{I_3}} = 0{,}145$$

$$\frac{w_{I_2}}{w_{I_3}} = 0{,}380$$

Where $\omega_{I_1}, \omega_{I_2}\ i\ \omega_{I_3}$ are the relative weights of the observed GDP variables, calculated using the Analytical Hierarchical Process. By identifying the minimum and maximum quotient, based on the type I insurance region, the lower and upper limits were determined, i.e., additional limitations of the model:

$$0{,}145 \leq \frac{v_{I_1}}{v_{I_2}}; \frac{v_{I_1}}{v_{I_3}}; \frac{v_{I_2}}{v_{I_3}} \leq 0{,}382$$

By solving the model with additional constraints, the results shown in Tables 8 -10 are obtained.

Table 8. Relative technical efficiencies of the RS national economy calculated using the CCR AR I model

Year	Score	Rank	1/Score	Reference set (lambda)	
2007	1	1	1	2007	1
2008	0.932078	2	1.072872	2007	1.018686
2009	0.0988	11	10.11709	2007	0.919736
2010	0.488608	8	2.04663	2007	0.950959
2011	0.548879	7	1.821895	2007	0.993761
2012	0.337385	9	2.96397	2007	0.898173
2013	0.721206	4	1.386566	2007	0.924378
2014	0.268826	10	3.719877	2007	0.826639
2015	0.554447	6	1.8036	2007	0.874473
2016	0.777833	3	1.285622	2007	0.883054
2017	0.685601	5	1.458574	2007	0.869251

Table 9. Optimal values of CCR AR weighting coefficients and models for assessing the technical efficiency of the RS national economy

Year	Score	v(1) I1	v(2) I2	v(3) I3	u(1) O1
2007	1	0.000064	0.000168	0.000441	0.10101
2008	0.932078	0.000067	0.000177	0.000465	0.106383
2009	0.0988	0.000704	0.001850	0.004850	1.111111
2010	0.488608	0.000138	0.000362	0.000949	0.217391
2011	0.548879	0.000117	0.000309	0.000809	0.185185
2012	0.337385	0.000211	0.000555	0.001460	0.333333
2013	0.721206	0.000096	0.000252	0.000662	0.151515
2014	0.268826	0.000288	0.000757	0.001990	0.454545
2015	0.554447	0.000132	0.000347	0.000910	0.208333
2016	0.777833	0.000093	0.000245	0.000642	0.147059
2017	0.685601	0.000107	0.000282	0.000740	0.169492

Table 10. Target values of input and output variables and the percentage change required to make DMUs efficiency

Year	Score	(I1) Data	(I1) Projection	(I2) Data	(I2) Projection	(I3) Data	(I3) Projection	(O1) Data	(O1) Projection
2007	1	51	0.00%	29.1	0.00%	2248.155	0.00%	9.9	0.00%
2008	1.072872	51.953	1.08%	29.644	-2.17%	2290.164	0.01%	10.0849	7.29%
2009	10.11709	46.907	-4.08%	26.764	37.96%	2067.709	-0.12%	9.10538	911.71%
2010	2.04663	48.499	-4.72%	27.673	49.58%	2137.904	-0.15%	9.41449	104.66%
2011	1.821895	50.682	-0.43%	28.918	43.87%	2234.128	-0.15%	9.83823	8.19%
2012	2.96397	45.807	-10.71%	26.137	24.46%	2019.232	-0.06%	8.89191	196.40%
2013	1.386566	47.143	-9.51%	26.899	52.84%	2078.144	-0.14%	9.15133	38.66%
2014	3.719877	42.159	-19.54%	24.055	37.46%	1858.413	-0.05%	8.18372	271.99%
2015	1.8036	44.598	-14.23%	25.447	34.64%	1965.951	-0.07%	8.65728	80.36%
2016	1.285622	45.036	-16.45%	25.697	34.54%	1985.242	-0.06%	8.74223	28.56%
2017	1.458574	44.332	-17.14%	25.295	20.45%	1954.212	-0.02%	8.60558	45.86%

4.4 Results of the DEA – AR model and discussion

The analysis of the obtained results shows that the introduction of additional restrictions related to the input variables affected the level of efficiency of the national economy of the RS in the observed period. In fact, in all years it is lower than that calculated in the less restrictive model without additional restrictions, but the movement by years was in a similar trend, one might say (Figure 2). Thus, in the new model, the efficiency was equal to 1 only at the beginning of the observed period, in 2007 (which is also the only reference year), which is identical to the result in the initial model, while in all other years it was greater than 1 (Table 10), in the output-oriented model, i.e. less than 1 (Table 9) in the input-oriented model. The lowest efficiency was achieved as in the previous model, in 2009, which coincides with the financial crisis that is, approximately, in that year It is also evident that the GDP growth rate in that year was the lowest in the observed period, and amounted to - 3.1 %. Table 10 shows the necessary changes (percentage reductions of input variables and percentage increase of output variable), which the national economy should have done in the observed years, in order to be technically efficient. On the other hand, it is visible (Table 9) that the optimal values of all weight coefficients are positive, so that the primary goal of the newly formed CCR model with the insurance region was achieved. In that sense, it seems that the second model, with additional limitations, more objectively reflects the reality and takes into account the fact that at no time can exogenous factors and their impact on the gross domestic product of a country, not even Serbia, be ignored.

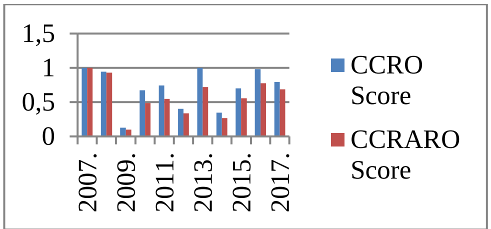

Figure 2. Comparative presentations of relative technical efficiencies of the national economy of RS using CCR-O and CCR AR models

Due to the sensitivity of the results obtained using the DEA method on measurement errors and outliers, the robustness analysis of the obtained efficiency values was performed, using the bootstrapp DEA (Efron, 1979; Simar and Wilson, 1998, 2000). As shown in Table 11, the average efficiency rating of the bootstrap is 0.580, which is almost identical to the original average efficiency (0.583). In addition, the bias, as the difference between the original average efficiency and the bootstrap efficiency estimate, is only 0.3%. Similar conclusions apply to the standard deviation. Finally, the average efficiency is included in the 95% confidence interval for the bootstrap efficiency score. That is, with 95% it can be claimed that the efficiency by years in the observed period was between 42.6% and 73.9%, and the confidence intervals to a large extent, overlap (Table 11). In this way, the consistency of the obtained DEA results was confirmed.

Table 11. Original and bootstrapping score values

Estimates	Score
Min	0.0988
Max	1.000
Mean	0.583
Bias	0.003
Mean (bootstrap)	0.580
Std.dev	0.277
Bias	0.017
Std.dev (bootstrap)	0.260
Lower bound (Mean)	0.420
Upper bound (Mean)	0.739
Lower bound (Mean bootstrapped)	0.433
Upper bound (Mean bootstrapped)	0.727
Lower bound (Std.dev bootstrapped.)	0.164
Upper bound (Std.dev. bootstrapped)	0.349

5. Conclusion

According to the results obtained by applying the standard output-oriented DEA CCR model, the national economy of the Republic of Serbia was efficient in the observed period only in 2007 and 2013. The average efficiency in the observed period was only 70% of the best practice. Although in the pre-crisis period, from 2001-2008, a relatively high average annual GDP growth rate of 5.4% was achieved, the model of economic development was realized through the creation and use of GDP with an unfavorable structure and high inflow of foreign capital, (through direct and portfolio foreign investments), and later, with the outbreak of the crisis, by direct foreign borrowing. In the period from 2005 to the beginning of the crisis, in August 2008, the Republic of Serbia recorded a strong growth of exports and an increase in the value of the dinar. However, as early as 2009 (but somewhat later than in developed countries, as a result of the underdeveloped financial system), the consequences of the global economic crisis followed, so that a slight recovery in economic activity in 2010 was not enough to return the economy to the same level. 2008 year. After 2012, the Serbian economy emerged from a recession in which GDP growth occurred, while fiscal consolidation and structural reforms had the effect of improving the business environment and realizing larger investments. Although economic growth was not very dynamic, it changed the growth paradigm, as it was based on sustainable resources, exports and investment, which was accompanied by employment growth, especially in the services sector. Also, the decline in technical efficiency in 2014, to 34% compared to the reference year 2007, is a consequence of the dramatic decline in the GDP growth rate, from 5.9% in 2007, to -1.8% in 2014 year. The positive signals of the recovery of the Serbian economy at the beginning of 2014 year were stopped by the catastrophic floods in May and September 2014 (the total damage is estimated at 1.7 billion euros). The energy sector suffered the most damage (EUR 800mill). In addition to the energy sector, the decline in GDP of -1.8% was also contributed by the decline in the manufacturing industry of -1.4% and after a record export in 2013 (25.8%), a modest export growth of 1.5%. The introduction of additional constraints of the model, using the method of the Analytical Hierarchical Process, formed the so-called assurance region, which affected the narrowing of the efficiency limit, but in both cases, it turned out, 2007 was the year in which the national economy was technically efficient. Despite certain theoretical controversies and doubts regarding the use of the DEA method to measure the efficiency of the whole economy, we consider that the paper shows that the applied DEA model reflects the macroeconomic trends of RS, so future research could be conducted in the direction of predicting efficiency and in that sense, determining the optimal level of use of available national resources - labor, capital and energy consumption.

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Appendix

Table 1. The scale of the relative importance used in the AHP/ANP models.

Intensity of Importance	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective
3	Moderate importance	Experience and judgment slightly favor one activity over another
5	Strong importance	Experience and judgment strongly favor one activity over another
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2, 4, 6, 8	Mean values between two adjacent estimates	When compromise is needed
Reciprocals of above	A reasonable assumption	If activity i has one of the above nonzero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i

Source: Saaty, T., and Kearns, K., Analytical Planning: The Organization of Systems, The Analytic Hierarchy Process Series, Vol.IV, 1985, p 27.

Comparison matrix

In relation to the main goal - determining the relative importance of variables that affect GDP, if the first factor is equally important or more important than the second, you assign one of the numbers from the scale 1-9, and if the second factor is more important, you assign a reciprocal value -1/2 , 1/3, .., etc.):

Table 2. Factor pairwise comparison matrix

Name	Capital	Energy consumption	Labor
Capital	**1**		
Energy consumption		**1**	
Labor			**1**