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## MULTIVARIATE CONTROL CHART WITH VARIABLE DIMENSIONS FOR FLEXIBLE PRODUCTION ENVIRONMENTS

**Abstract:** *Multivariate statistical control usually applied in industries that operate in continuous production processes. Thus, the objective of this work is to present a procedure for statistical process monitoring in flexible environments, which have a finite production horizon and  $p$  correlated observed variables. A systematic review of the literature on control charts conducted. Subsequently, a model proposition built and a simulation process performed; Thus, a method was conceived, which then validated through the Monte Carlo simulation. As an application of the proposed method, a case study was carried out in a metal-mechanic company in southern Brazil, which operates in the vertical transport segment. The results demonstrated the efficiency of the control chart in identifying and signaling out of control points, having a behavior similar to the expected behavior of its performance measure, ARL. The research contributes to the proposition of a multivariate control chart with variable dimensions for flexible production environments.*

**Keywords:** *Hotelling  $T^2$  Multivariate Control Chart; Finite Horizon; Flexible Environments.*

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## 1. Introduction

New production strategies have introduced significant changes in current production systems. Methodologies such as Lean Manufacturing and Just-in-Time emphasize smaller inventories, flexible manufacturing and reduced manufacturing batches (Li et al., 2014; Korzenowski et al., 2015; Sullivan; Jones, 2002). Large-scale production replaced by manufacturing a variety of products in small quantities. According to Lane et al. (2001), the manufacturing process increasingly stimulated by market forces and customer needs and perceptions. It is resulting in the need for flexible, multi-product manufacturing for various consumer profiles.

Due to the global market, which considers customer satisfaction as a prime objective, quality has established as a critical competitive priority in the business world (Castagliola et al., 2014). Consequently, the need for control and actions to reduce the variability in quality characteristics of the products in production increases. Quality is inversely proportional to variability; therefore, as variability in the essential characteristics of a product decreases, product quality increases thanks to the predictability of what is being produced (Montgomery, 2009).

According to Castagliola et al. (2014), Statistical Process Control (SPC) is a collection of statistical techniques that provides rational management of a manufacturing process, enabling the final

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product of high-quality products. Among the statistical process control tools, control charts are the most used to identify changes. These charts considered strategic tools for monitoring quality characteristics (Neneset al., 2014; Sullivan & Jones, 2002). However, it is challenging to apply traditional control chart methods in today's production environments such as univariate charts due to small series and small production lots (Zhang & Liu, 2009; Korzenowski et al., 2015). This is because short productions do not provide sufficient data to accurately estimate process parameters such as mean, standard deviation and limits for control charts (Li et al., 2014; Crowder & Eshleman, 2001). The assumption of independence of the observed data required by univariate charts usually violated since, in most cases, the quality of an item is characterized by correlated variables that, under normal operating conditions, determine process performance, impacting directly on product variability (Khoo & Quah, 2003; Lane et al., 2001). Therefore, the composition of control charts in processes with limited production horizon and correlated variables becomes a challenge for professionals and academics (Li et al., 2014; Nenes et al., 2014; Fogliatto et al., 2012). Modern and appropriate statistical methods needed to support and improve quality improvement efforts (Zhang & Liu, 2009; Crowder & Eshleman, 2001; Korzenowski et al., 2015; Fogliatto et al., 2012).

The literature has presented a growing and a considerable number of articles on control procedures that are capable of solving violations of data independence assumption, normality assumption, and insufficient data. According to Bersimis et al. (2009), in industry, there are several situations in which simultaneous monitoring or control of two or more characteristics related to the quality process required. In these cases, Multivariate Statistical Process Control indicated, since this type of chart considers the correlation between (Mohammadi et al., 2010). This approach mainly based on the Hotelling

statistic (Khoo & Quah, 2003).

It observed that there is still a possibility and demand for the development of new approaches due to the need for this type of monitoring in specific business models. For example, companies in the metal-mechanical sector perform forming of various types of sheet metal by mechanical folding processes via CNC 1, turning these sheets into complex geometries employing mechanical bending.

These geometries will have consecutive application in assemblies assembled manually or by robotic process welding. At this juncture, the level of requirement with product execution as per design specification becomes high, since only part of the geometry, if performed incorrectly, directly impacts the joining of the parts, as well as interfering with the preparation of the components, parts in devices used for robotic welding. In this process, there is an essential correlation between the folded tabs in the formation of the final part geometry, thus requiring controls based on multivariate analysis.

Some researches stand out for presenting solutions based on multivariate analysis. Sullivan & Jones (2002) proposed a Multivariate Cumulative Sum (CUSUM) control chart and a Multivariate Exponentially Weighted Moving Average (MEWMA) self-starting control chart that begins to monitor a  $p$ -variable process from observation  $(p + 2)$ . Castagliola et al. (2013) propose a graph with Variable Sample Size (VSS). Nenes et al. (2014) investigate the implementation of the Shewhart Control Chart with Variable Sampling Interval (VSI) in a process with a finite production horizon. Jarrett & Pan (2007) suggested a Vector Autoregression (VAR) Control Chart for multivariate autocorrelated processes. Korzenowski et al. (2013) propose the use of endogenous variables in predictive models to overcome the problems of multiple configurations and small productions encountered in custom manufacturing

systems. In addition, Vidor et al. (2015) proposes a method for determining which mass customization (MC) characteristics should prioritize in mass custom service design. While, Korzenowski & Werner (2012) verify the behavior of Shewhart's mean and standard deviation charts about the probability of type I error upon violation of the assumption of normality. Still, the implementation of process monitoring in manufacturing environments characterized as a challenge in which different types of sheet metal forming, which transformed into a variety of products with complex geometries through the configuration of various equipment setups. These types of products require a more flexible process in which it is not possible to form a data set long enough to estimate process parameters. According to Korzenowski et al. (2015), the key is to identify an approach to monitoring that implemented in a multi-item production environment, which means high flexibility and small batches. In this context, given the knowledge about the environment under study, it was not found in the literature, references on how to monitor a multivariate process applied to flexible environments, which implies the variation of the size of the monitoring vector, that is, the number of variables in analyze.

Thus, the objective of this paper is to present a procedure for statistical process monitoring in flexible environments, which have a finite production horizon and  $p$  correlated observed variables. The case study was conducted in a metal-mechanic company in southern Brazil that operates in the vertical transport segment, after the validation of the method by simulation. In addition to the introduction, the article structured as follows: Section 2 presents the research method; Section 3 shows the results of the systematic review of the multivariate statistical control chart literature for flexible environments and finite horizons; Section 4 presents the proposed model, followed by the results obtained by simulation to validate the method. Section 5 details the case study

with the application of the method and Section 6 presents the final considerations.

## 2. Method

The research was conducted through analysis for systematic, qualitative, and quantitative description of the manifested content of the literature. According to Hachicha & Ghorbel (2012), applying the principles of systematic review helps to limit bias (systematic errors), reduce the effects of chance, increase the legitimacy and authority of the following evidence, and provide results. Reliable decisions on which conclusions will draw and decisions.

Based on the method and aiming to ensure repeatability and scientific rigor, the following procedures adopted in the search for articles. The databases searched were the Web of Science and Science Direct. The databases selected according to Peres & Fogliatto (2018), as they are the most relevant databases in the Quality Control field of study. We only considered articles returned in the search with the following search key: (“statistical process control” OR “Control Chart” AND “self-start” OR “finite horizon” OR “short run”). We analyzed the articles that contained the keywords in the title or abstract. The operators “AND” and “OR” were used to ensure all word combinations.

Only articles published in scientific journals are considered. Search in the databases was not restricted to a specific period. Exclusion criteria were: (i) repeated articles and (ii) articles that did not mention multivariate monitoring of quality characteristics and therefore were not relevant to the theme. One of the authors of the present study read the titles and abstracts to apply the exclusion criteria. The articles kept were those in which there was an agreement between the two reviewers, researchers on the subject. These articles specifically addressed multivariate processes in short runs. The final group of articles thoroughly read, so

that the results could do classified and discussed. In the two databases consulted, 235 articles found in total. Twenty-eight articles excluded due to duplicity and 116 for lack of alignment with the theme, since these papers did not deal with the use of multivariate methods or did not present applications of these methods in the industrial context, dealing with applications in environments such as hospitals, services, or others. The titles and summaries of the

remaining 91 papers did thoroughly read and individually assessed by two specialists. Thus, 12 articles maintained, in which both evaluators agreed on the relevance of the theme and the potential contribution to the construction of the proposed model. The procedure used to select articles can see in Figure 1. The selected articles classified and characterized according to the criteria presented in Table 1, adapted from Hachicha & Ghorbel (2012).

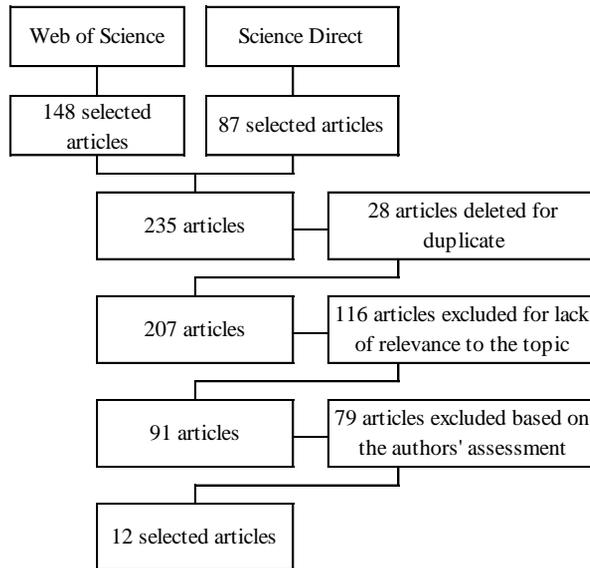
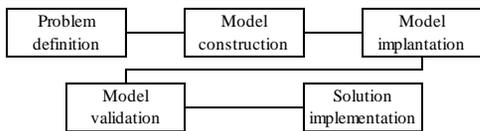


Figure 1. Article Search Steps

Table 1. Criteria for article classification

I. What are the assumptions of the data model?
(I.1) Non-standard distribution data
(I.2) MultivariateProcess Data
(I.3) Multivariate and autocorrelated process data
II. What is the input representation technique (data filter processing)?
(II.1) Based on raw data
(II.2) ResourceBased
III. What is the approach for detecting control chart signals?
(III.1) Rule-based system
(III.2) Others (Artificial Neural Networks, Hybrids or integrated or combined approaches, etc.)
IV. What is the type of validation of the approach?
(IV.1) Simulated data
(IV.2) Real process data
V. Quais são os critérios de sentido adotados?
(V.1) Average Run Length (ARL)
(V.2) Others

The second part of the research focused on modeling the monitoring problem that considers a flexible manufacturing environment with a finite production horizon, and products with correlated characteristics, core conditions observed in the examined production environment. According to Morabito et al. (2018), the use of models makes it possible to understand the environment in question better, identify problems, formulate strategies and opportunities, and support and systematize the decision-making process. Figure 2 presents the sequence of modeling and problem validation steps described in the following sequence.



**Figure 2.** Problem Modeling and Validation Steps

In the problem definition phase, the scope, the decisions of interest, the objectives involved, and the conceptual model are determined, according to Morabito et al. (2018). As observed in the production environment under study, interferences were identified in the stability of the bending sheet metal bending process, caused by variables such as increased frequency of setups, the variability of raw materials, increase in operational adjustment, etc. The objective is to create a tool that enables statistical process control, comprising a flexible production environment with correlated quality characteristics, since it has not happened identified in the literature a method that satisfies this type of treatment due to the numerous types of folds originated by different products.

In order to construct the model, the articles analyzed to determine the most appropriate strategies for process monitoring in flexible environments, where the product family lots

are small and varied, and each characteristic examined is dependent. These properties observed based on daily monitoring over a given period in order to understand the behavior of the process. Consequently, based on the articles and on-site analyzes, the method presented in section 4 of this article was defined.

According to Morabito et al. (2018), in the model implementation phase, solution methods and algorithms used for this purpose. These algorithms can do developed mainly to deal with the model exposed in the second phase. Therefore, an algorithm developed for solution and model definition through simulation, projecting the real system behavior over time.

Model validation performed by applying the Monte Carlo simulation technique. According to Khoo (2015), the Monte Carlo simulation method recommended as it provides time and cost savings while being more flexible in calculating the probabilities for different combinations of detection rules. Type 1 errors and false alarm rates will do checked, including whether ARLs are compatible with the type of data generated. With the tested and validated model, the behavior of the ARL will do evaluated, how many deviations are needed to detect special causes, and how quickly, as well as other attributes needed.

With the validated model, the actual process data tested. Two hundred samples collected every thirty minutes. Each inspected dimensionally of the product refers to an observed characteristic. The data collected, ie, the measured dimensions of the parts recorded using a standard data collection document. The geometry configurations of the products ranged from two to five folded tabs. A pre-processing of the data performed, enabling the organization of the products by categories whose number of fold tabs were equal, characterizing similarity of geometry.

### 3. Multivariate Statistical Control Chart

Univariate methods assume that there is only one output variable or quality of interest characteristic (Montgomery, 2009). However, in most current industrial applications, several related variables are involved, which are not independent of each other (Montgomery, 2009; MacGregor & Kourti, 1995). Therefore, multivariate methods required that consider the variables together, treating the data in parallel and extracting information about the process variability caused by the behavior of one variable relative to another (Montgomery, 2009; MacGregor & Kourti, 1995). Such

techniques called multivariate statistical process control procedures: a set of advanced techniques for monitoring and controlling the operational performance of continuous and batch processes that handle many highly correlated process variable measurements (Montgomery, 2009; Lowry & Montgomery, 1995). According to MacGregor & Kourti (1995), product quality defined only by the correct simultaneous values of all measured properties, ie, it is a multivariate property. For the classification of articles, the methodology of Hachicha & Ghorbel (2012), adapted to the multivariate statistical control chart used. Table 2 shows an overview of the classification according to the criteria presented in Table 1.

**Table 2.** Classification of selected articles

Articles by authors	I			II		III		IV	
	1	2	3	1	2	1	2	1	2
Li et al. (2017)	X	X			X			X	X
Korzenowski et al. (2015)		X		X			X	X	
Nenes et al. (2014)				X		X			X
Celano et al. (2013)				X		X			X
Zou et al. (2012)	X	X		X			X	X	
Li & Pu (2012)				X		X		X	X
Celano et al. (2011)				X		X			X
González & Sánchez (2009)		X		X		X			
Spiring (2008)					X		X		
Jarrett & Pan (2007)			X	X		X		X	
Wang (2005)		X		X			X		

Most articles focus on monitoring the average process vector through a multivariate method. Usually, works on multivariate processes use an approach through Hotelling's  $T^2$  graph. This graph has relative insensitivity to small and moderate changes in the process mean vector. Thus, the articles show the use of Multivariate Exponentially Weighted Moving Average (EWMA) Control Chart and Cumulative Sum (CUSUM) Control Chart, aiming at a better detection of changes.

Some processes do not have a normalized distribution. The assumption of non-normality of data is identified in Li et al.

(2017) and Zou et al. (2012), which proposes a bootstrap control plot that integrates a multivariate spatial classification test with the EWMA plot scheme based on variable selection to monitor sparse mean offsets.

Only the Vector Autoregression (VAR) Control Chart, based in ARL, proposed by Jarrett & Pan (2007) deals with autocorrelated multivariate data. The procedure uses a transformation of unknown parameter vectors into known parameter vectors of the same dimensionality through an exponentially weighted moving average graph with multivariate initialization.

Compared to the use of raw data from the process, it was little observed in the articles, the application of data preprocessing techniques. Regarding data preprocessing, a method is identified in Li et al. (2017), which proposes a spatial classification MEWMA Control Chart to transform the original process data.

Most selected jobs use rules to identify out-of-control process status, and the most widely adopted performance criterion is the calculation of the ARL. In Celano et al. (2011), Li & Pu (2012) and Nenes et al. (2014), some other ways of evaluating the performance of the control method are verified, such as Truncated Average Run Length (TARL), Average Number of False Alarms (ANFA) and Truncated Average Time to Signal (TATS). The articles divided as to the validation of the approached data. Half of the works use simulated data, such as Celano et al. (2011), which simulates a machining process. The other half uses real data, as is the case with Li et al. (2017) and Korzenowski et al. (2015). In the first, it is a process of producing white wine; The second section presents an example of data collected from a real plastics manufacturing environment located in southern Brazil.

Finally, articles dealing with the capability of multivariate processes classified, such as Wang (2005), who develops a procedure for the construction of Multivariate Process Capability Indices (MPCI) applied to short-term productions, using a principal component analysis technique.

#### 4. Multivariate control chart of variable dimensions for flexible production systems

Hotelling established multivariate process control techniques in his pioneering 1947 article, so the multivariate process control based on Hotelling's  $T^2$  Control Chart, which monitors the average process vector (Bersimis et al., 2009; Montgomery, 2009). The multivariate statistical control chart

requires preprocessing that consists of the standardization of each variable observed in each product. The adaptation required for bootstrapping as well as the monitoring of multiple products with varying dimensions follows Equation (1).

$$T_t^2 = Z'_{i,t} \hat{\Sigma}_i^{-1} Z_{i,t} \quad (1)$$

In Equation(1),  $Z_i$  corresponds to the vector of the standardized observed variables of the quality characteristics  $p$ , where  $i = 1, 2, \dots, m$ , where  $m$  is the number of products in the process,  $Z_{i,t}$  given by Equation (2) and  $\hat{\Sigma}_i$  is the covariance matrix comprising the variances and covariances of the standardized observed variables of product  $i$ .

$$Z_i = \frac{\varepsilon_{i,j}}{\hat{\sigma}_{i,j}} \quad (2)$$

where

$$\hat{\sigma}_{i,j}^2 = \sum_{i=1}^{n_i} \frac{\varepsilon_{i,j}^2}{n_{i-1}}$$

$\varepsilon_{i,j}$  is the observed difference of the quality characteristic  $j$  of product  $i$  from the value specified in design, with  $j = 1, 2, \dots, p$ . In the first phase of applying the Hotelling  $T^2$  graph, from individual observations, the control limit follows a Beta distribution according to Equation (3) (Montgomery, 2009).

$$L_c = \frac{(n_i-1)^2}{n_i} B_{1-\alpha/2; p/2; (n_i-p_i-1)/2} \quad (3)$$

In Equation (3),  $B_{1-\alpha/2; p/2; (n_i-p_i-1)/2}$  represents the percentile of a beta distribution with parameters  $p/2$  and  $(n_i - p - 1)/2$ , where  $n_i$  corresponds to the number of product  $i$  samples observed during Phase 1. Since in a flexible environment, the volume of production of each item (production on demand) not controlled, these sample sizes can be distinct, generating a variable control limit.

In phase 2, Hotelling  $T^2$  chart uses the distribution  $F$  for process monitoring, and the calculation of this limit given by Equation (4) (Montgomery, 2009).

$$L_c = \frac{(m+1)(m-1)p}{m(m-p)} F_{(p,m-p)} \quad (4)$$

In Equation (4),  $m$  is the number of subgroups of size  $n = 1$  observed in each product,  $p$  is the number of monitored variables in the product concerned and  $F$  is the quantile of the cumulative distribution  $F$  of Snedecor with  $1-\alpha$  probability.

### 5. Results

The process, object of this study, was performed in equipment that performs bending of sheet metal by applying a hydraulic force perpendicular to the flat plate. The same equipment is used to bend various types of geometry and approximately 26 different raw material types, ranging from 2mm to 9.52mm in thickness. On average, 224 different product families produced on the horizon of one month, including

standardized and customized items. The machine has two work shifts, with one operator on each shift. The objective was to monitor the stability of this process. For this, from March 14 to August 15, 2018, every half hour, measurements of various types of work were performed using a measuring instrument available at the workplace, comprising its two work shifts. In total, due to their finite production horizon, 200 samples were collected ( $n = 1$ ). Due to the high process variability, the dimensions of 94 different products observed. Since each tab dimension corresponds to one variable, a multivariate monitoring approach is warranted, and the folded profile model ranges from two to five fold tabs, as shown in Figure 3.

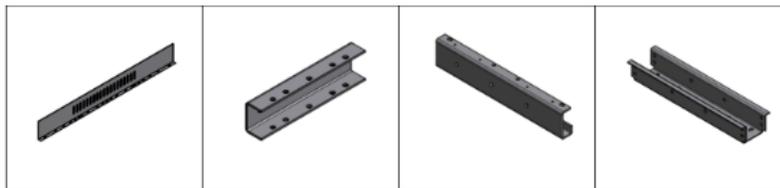


Figure 3. Folded Profile Types

The standard deviations of each dimension found concerning the nominal design measure calculated. Thus, it was possible to calculate the standard error and to group the various products by type of bending model. The proposed standardization made it possible to segregate the various products by model type, defined by the number of tabs that make up this model (profile). Table 3 shows the number of samples collected per profile.

The covariance matrices were calculated for each profile and then used to calculate the Hotelling  $T^2$  statistic. Figure 4 shows the  $T^2$  control chart for flexible environments with variable dimensionality. In the figure, it is possible to identify that the value of 13 samples from the 200 collected exceeded the control limits, causing the possible influence of attributable causes in the process. Control

limits vary according to the size of samples taken from their respective families (profile).

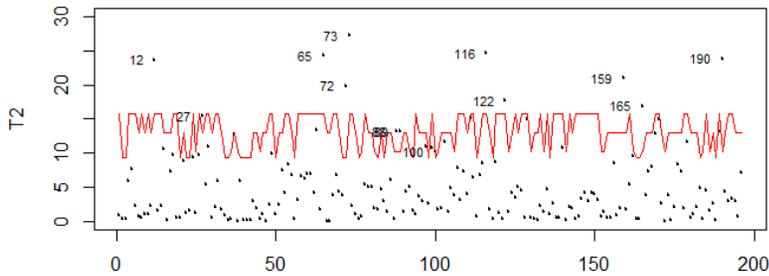
Table 3. Process Flexibility

Profile	n	# Products by profile
2	29	17
3	68	30
4	24	9
5	79	38

Figure 3 presents evidence of the dimensional variety and complexity level of shaped products in the environment in which the process allocated. As a result, the process is vulnerable and sensitive to a variety of causes, such as operators undergoing training, complex geometries, equipment deregulated by setup frequency, different raw materials, and others. The process variability, reflecting these interferences, is expressed in Figure 4, since, as can be

observed in samples 12, 65, 72, 116, 165 and 190, which exceeded the control limits due specifically to unfolding of the cut blank (dimensional of the flat plate), customization and displaced parts of its conventional process. Observations 27, 73, 89, 100, and

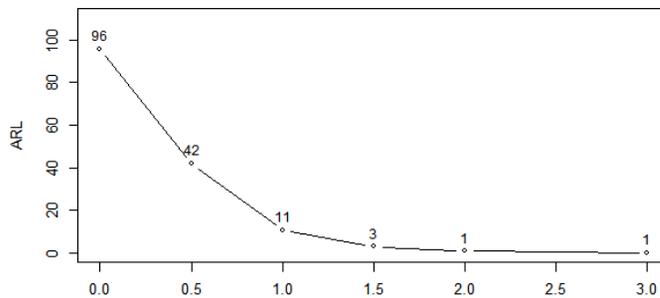
159 presented errors due to very complex product geometries, leading from errors in the incorrect execution of the design dimension to measurement errors. The remaining out-of-control samples (88 and 122) considered false alarms.



**Figure 4.** Phase 1 of the profile bending process monitoring

A control chart needs several steps to be developed; through a model, it must be validated and evaluated. For practical and economic reasons, these steps performed through computer simulation experiments. Simulation environment tests allow the evaluation of different scenarios and model configurations in order to evaluate their robustness against the possible cases faced in a real production environment (Law & Kelton, 1991). Therefore, the control chart of the present work validated through the Monte Carlo simulation, which reproduced the variability in product characteristics at random, following the proportion of items

based on the actual historical data set. A flexible process simulated with 4 product families: A, B, C, and D. These had 2, 3, 4, and 5 critical quality characteristics that needed monitoring. Process data such as sample size, standard deviations of quality characteristics, and covariance matrices were estimated and used for the generation of the simulated data in Phase 2. ARL deviations generated in the order of 0, 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0 standard deviations from the mean of the variables. Figure 5 presents the graph of the behavior of the ARL found in the simulation process.



**Figure 5.** ARL obtained in the simulation by deviations from the average

The model has its performance evaluated when subjected to different circumstances. For this, five thousand series were generated,

which, on average, are not out of control. A false alarm is generated for every 96 observations, as shown in the ARL chart. If

the process is impacted by some cause, such as a variation in the equipment, causing the average to shifted upwards, the graph will take an average of 42 observations to detect this shift. From the change of two standard deviations, the graph needs only one observation to detect and alarm. According to Li et al. (2017), Statistical Process Control Multivariate charts are useful tools for monitoring various quality characteristics simultaneously, and their performance usually measured by the Average Run Length (ARL). In this case, the model demonstrates an expected performance, given the decreasing curve shown in the graph (ARL). Therefore, the graph is useful in detecting changes in process parameters.

## 6. Conclusion

As a result of the research, it was proposed to present a statistical procedure for process monitoring in flexible environments, due to the lack of adequate methods in the literature. Therefore, a solution based on Hotelling's  $T^2$  statistics was proposed, which through standardization of the data allowed to treat the monitoring situation in an environment where there is a lot of product variability, presenting diverse geometries correlated finite production horizons. Through the results, it observed that the data behavior, about the process deviations, followed the expected. The procedure, in its application in the real case, was sensitive to the causes of variability examined daily in the process, satisfactorily reflecting the process deviations often identified in parts manufacturing, as out-of-control points justified based on special causes. A small

number of points are false alarms, following the simulation result, in which the expected process behavior signals approximately two false alarms for each sample size 200.

The absence of an appropriate method to deal with this type of case made it difficult to implement the letter in a productive environment. Although the procedure has proved effective, there are still many challenges to its implementation in industrial reality. There is no defined way to perform data collection and online monitoring, and there is a need for information technology development support, which will suffer commercial implications due to the complexity of the calculations to be performed. This situation requires the corporation to invest more in the development of a system where the machine operator can only enter the measured data, and it can be viewed in an engineering background, setting up remote monitoring from information-fed online, directly at the factory. Resources can also do deployed in technologies that promote the automatic measurement process by automating inspection, facilitating monitoring, and speeding detection of attributable causes. For the metal-mechanic production environment, considering the shrinking production batches, it is also relevant to investigate a study based on a self-starting multivariate statistical control chart, which discards the need for phase I data collection. It is necessary to develop and build multivariate capability indices for the process analyzed in this study in order to stabilize the process and ensure full compliance with project characteristics.

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