International Journal for Quality Research 13(3) 553–574
ISSN 1800-6450

Divya T. Patel
Manharlal N. Patel [1]

Article info:
Received 03.08.2018
Accepted 17.03.2019

UDC – 330.133.1
DOI – 10.24874/IJQR13.03-04

A BAYESIAN APPROACH TO THE OPTIMAL WARRANTY LENGTH FOR PARETO DISTRIBUTED PRODUCT WITH THE GENERAL PROGRESSIVE TYPE-II CENSORING SCHEME

Abstract: *The object of the study is to determine the optimal warranty length under free replacement warranty (FRW), pro rata warranty (PRW) and combined warranty policies and the most beneficial warranty scheme to the producer for the product having Pareto life time distribution. A Bayesian approach is used to determine the optimal warranty length based on the general progressive type-II censored data. The optimal warranty is obtained by maximizing the expected utility of the product. A numerical data is presented to exemplify the theory. A simulation study is carried out to check the effect of the hyper parameters on the optimal warranty length and the optimal value of expected utility. From our study we observed that the combined policy gives maximum utility followed by PRW and then by FRW for any choice of the prior parameters. Hence we suggest the producer to adopt the combined policy for such a product.*

Keywords: *Posterior distribution, warranty policy, economic benefit function, warranty cost function, dissatisfaction cost function, general progressive type-II censoring scheme*

1. Introduction

The manufacturers may attract consumers to purchase their products by providing reasonable warranties on the products with the major goal of increasing profits. To increase the profit the important factors are sale volume and the selling price. Sale volume of the product depends not only on the lower price of the product but also depend on the on the quality, reliability and warranty length of the product. A good quality product requires some more cost, which increases the selling price of the product (Scitovszky, 1945). To reduce the selling price producer may produce the product in a very large quantity. To compete with standard product producer should produce the products having good quality and competitive price to fulfill customers expectations. Determination of the appropriate selling price of the product is also an issue for the producer. Jeyakumar and Jevakumar and Robert (2010) considered joint determination of warranty length as well as production quantity under free renewal policy. Quality of the product can be judged by its types warranty and warranty length. Warranty is a contract between the manufacturer and a customer that gives assurance to the customer about the quality of the product. Through warranties, customers are provided guarantees for completely free replacement of the product or partial replacement, even in terms of money for a period of time following the purchase of

[1] Corresponding author: Manharlal N. Patel
Email: mnpatel.stat@gmail.com

product. Thus, a proper warranty plays an important role in increasing sales as well as profit from the products. Such type of work has been done by many authors like Singpurwalla and Wilson (1998). If the manufacturers wish to give compensation to the buyer when failure occurs, the warranty length and the reliability of the product play a significant role on determining the cost of the product. Optimal warranty length in case of the product possessing Rayleigh distributed life time is considered by Wu and Huang (2010). As the Rayleigh distribution has an increasing failure rate over a time, such a study will not be useful for the product having constant or decreasing failure rate. Wu et al. (2006[a]) have considered normal distribution as a product life time model which is suitable only for the product having increasing failure rate. Life time of the product may follow various types of life time distributions like Exponential, Power function, Kumaraswamy distributions. Patel and Patel (2017) have considered a Bayesian approach to optimal warranty length for a Kumaraswamy life time distributed product with general progressive censoring scheme. In this paper we have considered power function life time model for the product having decreasing failure rate.

The knowledge of product reliability is must for a manufacturer to design a cost-effective warranty. Such a knowledge about the reliability of the product can be acquired by conducting life testing experiment. Since the life testing experiments are destructive, which increases the expenses of a producer. To save time and cost censored experiments are conducted. Usually two basic types of censoring schemes are used in life testing experiments. Type-I censoring and Type-II censoring are the most commonly used censoring schemes. Such censoring schemes have been studied by number of authors including Lawless (1982), Gouno et al. (2004), Balakrishnan et al. (2007). There is no facility to withdraw some units, which may be useful for any other purpose, from the survival units during the experiment before the final termination of the test. There are some censoring schemes which allow such type of withdrawal, like progressive type-I, progressive type-II, general progressive type-II, progressive first failure or multiply type-II censoring schemes. Nadi and Gildeh (2016) considered progressive first-failure censoring scheme to estimate the life time performance index for two-parameter exponentially distributed life time product. In this paper we use general progressive type-II censoring in which some failure units are withdrawn from the test.

The aim of this paper is to determine optimal warranty length for the product having Pareto distribution. The information of product reliability is obtained through a general progressive type-II censored life test. The utility function and information are used to determine the warranty length under Bayesian set up. The concept of utility function to determine optimal warranty period as considered by Wu and Huang, (2010) is used. In section 2 the likelihood function for the Pareto distribution is constructed based on the general progressive type-II censored sample. Using gamma conjugate prior distribution for the parameter of the life time distribution, the posterior distribution is obtained. A posterior predictive distribution is derived using the posterior distribution. Section 3 gives the warranty policies. A combined warranty policy based on FRW (Free replacement warranty) and PRW (Pro-rata warranty) are described. Cost functions under the above warranty policies are mentioned. Section 4 provides utility function which is constructed using economic benefit function, warranty cost function and dissatisfaction cost as described by Wu and Huang (2010). Section 5 covers the maximization of expected utility function and optimal warranty. Section 6 provides a numerical example. The sensitivity analysis is also carried out in Section 7 to study the effect of the prior parameters. Some conclusions are drawn in Section 8.

2. Life time model and posterior distribution

The Pareto distribution has its own importance in the life testing experiments. This distribution has been considered by many authors like Aggarwala and Childs (1999), Hossain and Zimmer (2000), Mahmmad et al. (2013), Podder et al. (2004), Shah and Patel (2007) as a life time model.
The probability density function of Pareto distribution is given by

$$f(x \mid \theta) = \theta\, x^{-\theta-1}, \; x \geq 1, \; \theta > 0 \qquad (1)$$

Its cumulative distribution function is given by

$$F(x \mid \theta) = 1 - x^{-\theta}, \; x \geq 1, \; \theta > 0 \qquad (2)$$

Hence the failure rate of the distribution becomes

$$h(x) = \frac{\theta}{x}, \; x \geq 1, \theta > 0 \qquad (3)$$

It is very common that the lifetimes of some test units may not be able to be recorded exactly. For example, in type-II censoring, the test ceases after a predetermined number of failures in order to save time or cost. Moreover, some test units may have to be removed at different stages in the study for various reasons this would lead to a progressive censoring. Progressive Type-II censoring is an important method of obtaining data in lifetime studies. Live units removed early can be readily used in other tests, thereby saving costs to the experimenter. In Statistical inference progressive censoring has received the attention of many authors. Articles by Cohen (1963), Mann (1971), and Viveros and Balakrishnan (1994), Wu et al. (2006[b]) Gajjar and Patel (2008), Patel and Patel (2007), are of some early works on estimation under progressive censoring. Blakrishnan and Sandhu (1996) considered the general progressive censoring scheme to obtain best linear unbiased and maximum likelihood estimator of the parameter of exponential distribution. /in this paper we have used such a censoring scheme to determine posterior predictive density function based on Bayesian setup.
Suppose n units were placed on a life test and first r failure times $Y_1,\ldots,Y_r$ are not observed. At failure time Y_{r+1}, R_{r+1} units are removed randomly form the survival units on the test, at failure time Y_{r+2}, R_{r+2} units are removed randomly form the survival units on the test and so on. Finally, experiment is terminated at the m[th] failure at failure time Y_m with remaining R_m survivals. Therefore, $Y_{r+1} \leq \ldots \leq Y_m$ are the lifetimes of the completely observed units to fail and there are n_i units on test at (i+1)[th] failure where

$$n_i = n - i - \sum_{j=r+1}^{i} R_j, \; i = r+1, \ldots, m-1. \qquad (4)$$

Here $R_{r+1}, R_{r+2}, \ldots, R_m$ are fixed numbers predetermined by the experimenter. The general form of the likelihood function based on the above described general progressive type-II censoring is given by:

$$L(\theta, x) = \frac{n!}{r!(n-r)!}\left(\prod_{j=r}^{m-1} n_j\right)\left[F(t_{r+1})\right]^r \prod_{i=r+1}^{m} f(t_i,\theta)\left[1 - F(t_i)\right]^{R_i} \qquad (5)$$

Using probability density function and cumulative distribution function from (1) and (2) we have the likelihood function as

$$L(\theta, x) = c\theta^{m-r}\left(1 - x_{r+1}^{-\theta}\right)\prod_{i=r+1}^{m}\left(x_i^{-\theta R_i}\right)\left(\prod_{i=r+1}^{m} \theta x_i\right)^{-\theta-1} \qquad (6)$$

where

$$c = \frac{n!}{r!(n-r)!}\prod_{j=r}^{m-1} n_j \tag{7}$$

To obtain posterior distribution of parameter θ, here we use the gamma conjugate prior for θ as given by

$$\pi(\theta) = \frac{\delta^{v}}{\Gamma v}\theta^{v-1}e^{-\delta\theta}, \theta > 0, v > 0, \delta > 0 \tag{8}$$

Here the posterior distribution of the parameter θ can be obtained as

$$\pi(\theta \mid x) = \frac{L(\theta, x)\,\pi(\theta)}{\int_{\theta} L(\theta, x)\,\pi(\theta)\,d\theta} = \frac{\theta^{m-r+v-1}\sum_{j=0}^{r} h_1(j) e^{-\theta(j\log x_{r+1} + A_{rm} + \delta)}}{\Gamma(m-r+v)\sum_{j=0}^{r}\frac{h_1(j)}{(j\log x_{r+1} + A_{rm} + \delta)^{m-r+v}}} \tag{9}$$

where

$$A_{rm} = \sum_{i=r+1}^{m} \log x_i (R_i + 1)$$

$$h_1(j) = (-1)^{-j}\binom{r}{j}, j = 0,1,\ldots\ldots,r \tag{10}$$

From (2.1) and (2.9) the posterior predictive distribution can be obtained using the result

$$f(t \mid x) = \int_0^{\infty} f(t \mid \theta)\,\pi(\theta \mid x)\,d\theta \tag{11}$$

as

$$f(t \mid x) = \frac{\sum_{j=0}^{r} h_1(j)\frac{(m-r+v)}{(j\log x_{r+1} + A_{rm} + \delta + \log t)^{m-r+v+1}}}{t\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta + \log t)^{m-r+v}}} \tag{12}$$

Hence the posterior predictive cumulative distribution function can be obtained as

$$F(w \mid x) = \int_1^{w} f(t \mid x)\,dt = \int_1^{w}\frac{\sum_{j=0}^{r} h_1(j)\frac{(m-r+v)}{(j\log x_{r+1} + A_{rm} + \delta + \log t)^{m-r+v+1}}}{t\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta)^{m-r+v}}}dt$$

which can be further simplified by taking

$$y = \ln t$$

$$F(w|x) = \frac{(m-r+v)\int_{0}^{\ln w}\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta + y)^{m-r+v+1}}dy}{\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta)^{m-r+v}}}$$

$$= \frac{\sum_{j=0}^{r} h_1(j)\begin{Bmatrix} -(j\log x_{r+1} + A_{rm} + \delta + \ln w)^{-m+r-v} \\ +(j\log x_{r+1} + A_{rm} + \delta)^{-m+r-v}\end{Bmatrix}}{\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta)^{m-r+v}}} \tag{13}$$

Now consider the integration

$$I_{11} = \int_{l_1}^{u_1} tf(t|\theta)\,\mathrm{dt}$$

$$= \int_{l_1}^{u_1} t\frac{\sum_{j=0}^{r} h_1(j)\frac{(m-r+v)}{(j\log x_{r+1} + A_{rm} + \delta + \log t)^{m-r+v+1}}}{t\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta)^{m-r+v}}}dt$$

$$= \frac{(m-r+v)\int_{l_1}^{u_1}\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta + \log t)^{m-r+v+1}}dt}{\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta)^{m-r+v}}}$$

$$= \frac{(m-r+v)I_0}{\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta)^{m-r+v}}} \tag{14}$$

where:

$$I_0 = \int_{l_1}^{u_1}\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta + \log t)^{m-r+v+1}}dt \tag{15}$$

3. Warranty Policy

Here we have considered a combination of the two commonly used warranty policies namely free replacement warranty and pro rata warranty. Under FRW policy, if a product fails during the warranty period, the product is replaced by another product of the same kind free of charge.

Under PRW policy the manufacturer gives compensation to the buyer on the basis of the failure time during the warranty policy, which may be a linear function of the remaining time of the warranty period.

A combination of these two types of policies is called combined FRW/PRW policy.

Here we assume FRW during the period $[0, w_1)$, and PRW during the period $[w_1, w_2)$, where $w_1 \le w_2$ are positive values. The reimbursing cost function of an item with time length t for combined FRW/ PRW policy is given by

$$C_W(t) = \begin{cases} S, & 0 \le t < w_1 \\ S\left(\dfrac{w_2 - t}{w_2 - w_1}\right), & w_1 \le t < w_2 \\ 0, & t \ge w_2 \end{cases} \quad (16)$$

In case of FRW policy ($w_1=w_2$) the reimbursing cost function reduces to (17), and under PRW policy ($w_1=0$) the reimbursing cost function reduces to (18)

$$C_W(t) = \begin{cases} S, & 0 \le t < w_1 \\ 0, & t \ge w_1 \end{cases} \quad (17)$$

$$C_W(t) = \begin{cases} S\left(\dfrac{w_2 - t}{w_2}\right), & 0 \le t < w_2 \\ 0, & t \ge w_2 \end{cases} \quad (18)$$

where S is the selling price of the product which is cost to the buyer.

This cost function is also called the manufacturer loss associated with setting up a warranty.

4. Utility Function

In the combined FRW/PRW policy, the warranty length, say w_1 and w_2 are determined for a product. To determine the values of w_1 and w_2 one has to consider a function of warranty policy that measures the monetary utility when the product fails at time t.

Here we consider the utility function, used by Wu and Huang (2010) based on the economic benefit function $B(w_1, w_2)$,the warranty cost function $W(t, w_1, w_2)$ and the dissatisfaction cost function $D(t, w_1, w_2)$ defined as (19). The economic benefit function is proposed as (20).

$$U(t, w_1, w_2) = B(w_1, w_2) - W(t, w_1, w_2) - D(t, w_1, w_2) \quad (19)$$

$$B(w_1, w_2) = A_1 M \left(1 - e^{-A_2\left(\frac{w_1 + w_2}{2}\right)}\right) \quad (20)$$

where A_1 is the profit per product obtained by manufacturer and M is the potential number of products to be sold with this warranty policy. The parameter A_2 can be derived by solving the equation (21), which is the parameter to control the speed of increment in benefit.

$$\frac{B(0, t_W)}{B(t_W, t_W)} = \frac{1 - e^{-\left(\frac{A_2 t_W}{2}\right)}}{1 - e^{-A_2 t_W}} \quad (21)$$

The ratio shows the percentage of benefit remains when the manufacturer changes the warranty from FRW to PRW. The warranty cost function $W(t, w_1, w_2)$ is an item $C_w(t)$ times the expected number of items that fail under the warranty period. The expected number of failures can be determined using the method given by Wu and Huang (2010) based on the posterior predictive cumulative distribution function under the approach of trinomial distribution. Thus, the warranty cost function can be obtained as (22).

$$W(t, w_1, w_2) = MF\left(w_1|x\right)SI_{[0, w_1)}(t) + M\left[F\left(w_2|x\right) - F\left(w_1|x\right)\right]S\left(\frac{w_2 - t}{w_2 - w_1}\right)I_{[w_1, w_2)}(t) \quad (22)$$

where $I_{[a,\ b)}(t)$ is an indicator function which assumes the value one when a ≤ t < b, and zero otherwise.

The dissatisfaction cost is the manufacturer's indirect cost, when the product fails during the warranty period, or fails during time just after warranty, such cost function is used by Djamaludin et al. (1996).

Under the combined FRW/PRW policy we have used the dissatisfaction cost function considered by Wu and Huang (2010) as (23).

$$D(t, w_1, w_2) = D_1(t, w_1) + D_2(t, w_1, w_2) + D_3(t, w_2) \quad (23)$$

In case of FRW policy, when product fails in the time period $[0, w_1)$, the dissatisfaction cost is a proportion $q_1 (0 < q_1 < 1)$ of the sales price S, multiplied by the expected number of failures. i.e.

$$D_1(t, w_1) = M\ F\left(w_1|x\right)Sq_1\ I_{[0,\ w_1)}(t) \quad (24)$$

The second component is for the product fails during the time interval $[w_1, w_2)$. Here it is assumed the dissatisfaction cost of an item linearly decreases with time with maximum Sq_1 and minimum $Sq_2, 0 < q_2 < q_1 < 1$.
Hence,

$$D_2(t, w_1, w_2) = M\ \left[F\left(w_2|x\right) - F\left(w_1|x\right)\right] \times \left[Sq_1 - (Sq_1 - Sq_2)\left(\frac{t - w_1}{w_2 - w_1}\right)\right] I_{[w_1,\ w_2)}(t) \quad (25)$$

And the third component $D_3(t, w_2)$ is for the product fails after the expiration of warranty, but the customer may still be unsatisfied with the product unless its lifetime exceeds a specified value $L, L > w_2$.

Here $D_3(t, w_2)$ decreases linearly with time t, reaching to zero when lifetime is L and given by

$$D_3(t, w_2) = M\ \left[F\left(L|x\right) - F\left(w_2|x\right)\right] \times Sq_2\left(\frac{L - t}{L - w_2}\right)\ I_{[w_2,\ L)}(t) \quad (26)$$

The value of L may be considered as the mean or median or percentile of the posterior predictive distribution given in (12).

5. Optimal Warranty

The optimal warranty (w_1^*, w_2^*) is that which maximize the expected value of the utility function EU with expectation over the posterior predictive distribution,
That is

$$E(U(T, w_1, w_2)) = \int_0^{\infty} U(t, w_1, w_2)\, f(t|x)dt \quad (27)$$

Using the equation (19) and (12) in the above equation (27), we get the expression for the expected utility function as (28).

After some mathematical manipulation we get the expected utility function as (29).

$$E(U(T, w_1, w_2)) = \int_0^{\infty} \{B(w_1, w_2) - W(t, w_1, w_2) - D(t, w_1, w_2)\} f(t \mid x) dt \tag{28}$$

$$E(U(T, w_1, w_2)) = I_1 - I_2 - I_3 \tag{29}$$

where

$$I_1 = \int_0^{\infty} B(w_1, w_2) \, \mathrm{f}(\mathrm{t} \mid \mathrm{x}) dt \quad = MA_1 (1 - e^{-A_2 (\frac{w_1 + w_2}{2})}) \tag{30}$$

$$I_2 = \int_0^{\infty} W(t, w_1, w_2) f(t \mid x) dt$$

Using this formula we have

$$I_2 = M \begin{bmatrix} S[F(w_1 \mid x)]^2 - \\ S[F(w_2 \mid x) - F(w_1 \mid x)][I_{2.1} - I_{2.2}] \end{bmatrix} \tag{31}$$

$$= M \left[\begin{array}{l} S \left[\dfrac{\sum_{j=0}^{r} h_1(j) \left\{ \begin{array}{l} -(j \log x_{r+1} + A_{rm} + \delta + \ln w_1)^{-m+r-v} \\ +(j \log x_{r+1} + A_{rm} + \delta)^{-m+r-v} \end{array} \right\}}{\sum_{j=0}^{r} h_1(j) \dfrac{1}{(j \log x_{r+1} + A_{rm} + \delta)^{m-r+v}}} \right]^2 \\ - S \left[\dfrac{\sum_{j=0}^{r} h_1(j) \left\{ \begin{array}{l} -(j \log x_{r+1} + A_{rm} + \delta + \ln w_2)^{-m+r-v} \\ +(j \log x_{r+1} + A_{rm} + \delta + \ln w_1)^{-m+r-v} \end{array} \right\}}{\sum_{j=0}^{r} h_1(j) \dfrac{1}{(j \log x_{r+1} + A_{rm} + \delta)^{m-r+v}}} \right] [I_{2.1} - I_{2.2}] \end{array} \right] \tag{32}$$

where,

$$I_{2.1} = \int_{w_1}^{w_2} \left(\frac{w_2}{w_2 - w_1} \right) f(t \mid x) dt$$

$$= \left(\frac{w_2}{w_2 - w_1} \right) [F(w_2 \mid x) - F(w_1 \mid x)]$$

$$= \left(\frac{w_2}{w_2 - w_1}\right)\left[\left[\frac{\sum_{j=0}^{r} h_1(j)\begin{Bmatrix} -\left(j\log x_{r+1} + A_{rm} + \delta + \ln w_2\right)^{-m+r-v} \\ +\left(j\log x_{r+1} + A_{rm} + \delta + \ln w_1\right)^{-m+r-v}\end{Bmatrix}}{\sum_{j=0}^{r} h_1(j)\frac{1}{\left(j\log x_{r+1} + A_{rm} + \delta\right)^{m-r+v}}}\right]\right] \quad (33)$$

$$I_{2.2} = \int_{w_1}^{w_2}\left(\frac{t}{w_2 - w_1}\right) f(t|x)\,dt$$

$$= \left(\frac{1}{w_2 - w_1}\right)\int_{w_1}^{w_2} t\, f(t|x)\,dt$$

$$= \left(\frac{1}{w_2 - w_1}\right)\frac{(m - r + v)I_{12}}{\sum_{j=0}^{r} h_1(j)\frac{1}{\left(j\log x_{r+1} + A_{rm} + \delta\right)^{m-r+v}}} \quad (34)$$

Where,

$$I_{12} = \int_{w_1}^{w_2}\sum_{j=0}^{r} h_1(j)\frac{1}{\left(j\log x_{r+1} + A_{rm} + \delta + \log t\right)^{m-r+v+1}}dt \quad (35)$$

Now,

$$I_3 = M\begin{bmatrix} Sq_1[F(w_1|x)]^2 - S[F(w_2|x) - F(w_1|x)][I_{3.1} - I_{3.2}] - \\ Sq_2[F(L|x) - F(w_2|x)][I_{3.3} - I_{3.4}]\end{bmatrix} \quad (36)$$

where

$$I_{3.1} = \int_{w_1}^{w_2} q_1 f(t|x)\,dt = q_1\left[F\left(w_2|x\right) - F\left(w_1|x\right)\right]$$

$$= q_1\left[\left[\frac{\sum_{j=0}^{r} h_1(j)\begin{Bmatrix} -\left(j\log x_{r+1} + A_{rm} + \delta + \ln w_2\right)^{-m+r-v} \\ +\left(j\log x_{r+1} + A_{rm} + \delta + \ln w_1\right)^{-m+r-v}\end{Bmatrix}}{\sum_{j=0}^{r} h_1(j)\frac{1}{\left(j\log x_{r+1} + A_{rm} + \delta\right)^{m-r+v}}}\right]\right] \quad (37)$$

$$I_{3.2} = \int_{w_1}^{w_2}\left(\frac{q_1 - q_2}{w_2 - w_1}\right)(t - w_1) f(t|x)\,dt$$

Using we can get

$$= \frac{q_1 - q_2}{w_2 - w_1}\left[\int_{w_1}^{w_2} tf(t|x)dt - w_1\left[F(w_2|x) - F(w_1|x)\right]\right] \tag{38}$$

$$I_{3.2} = \frac{q_1 - q_2}{w_2 - w_1}\left[\frac{(m-r+v)I_{12}}{\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta)^{m-r+v}}} - w_1\left[\frac{\sum_{j=0}^{r} h_1(j)\left\{\begin{array}{l} -(j\log x_{r+1} + A_{rm} + \delta + \ln w_2)^{-m+r-v} \\ +(j\log x_{r+1} + A_{rm} + \delta + \ln w_1)^{-m+r-v}\end{array}\right\}}{\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta)^{m-r+v}}}\right]\right] \tag{39}$$

$$I_{3.3} = \frac{L}{L - w_2}\left[\left(F(L \mid x) - F(w_2 \mid x)\right)\right]$$

$$= \frac{L}{L - w_2}\left[\frac{\sum_{j=0}^{r} h_1(j)\left\{\begin{array}{l} -(j\log x_{r+1} + A_{rm} + \delta + \ln L)^{-m+r-v} \\ +(j\log x_{r+1} + A_{rm} + \delta + \ln w_2)^{-m+r-v}\end{array}\right\}}{\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta)^{m-r+v}}}\right] \tag{40}$$

and again using (14) we have

$$I_{3.4} = \frac{1}{L - w_2}\int_{w_2}^{L} tf(t \mid x)dt$$

$$= \frac{(m-r+v)I_{2l}}{(L - w_2)\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta)^{m-r+v}}} \tag{41}$$

where,

$$I_{2l} = \int_{w_2}^{L}\sum_{j=0}^{r} h_1(j)\frac{1}{(j\log x_{r+1} + A_{rm} + \delta + \log t)^{m-r+v+1}}dt \tag{42}$$

where I_{12} and I_{2l} are same as defined the integral in (7).

Using (30) to (36) in (28) we will get an expression for expected utility function.

Thus the optimal warranty (w_1^*, w_2^*) is given by the solution to the optimization problem.

$$(w_1^*, w_2^*) = \arg \left(\max_{w_1 \le w_2 \in R^+} E(U(T, W_1, W_2)) \right)$$

Where R^+ denotes the set of positive real numbers.

This is difficult to solve analytically but computer program can also be prepared to solve it.

6. Numerical example

To illustrate the theoretical results we consider the following example:

Let us assume the selling price of the product whose production cost is Rs. 175, fixed by the manufacturer is S=Rs. 250 so that the profit per product becomes$A_1 = Rs. 75$. We further assume that the
manufacturer fixed the proportions of loss from consumer dissatisfaction for timeperiod $[0, w_1)$ as $q_1 = 0.2$ and for time period $[w_1, w_2)$ as $q_2 = 0.1$. Suppose that the life time of the product follows Pareto distribution given in (1).

The life times of such 15 products, generated by taking θ =2 are given below:

1.019332,1.140674,1.165424,1.183377,1.212933,1.325606,1.423381,1.426641,1.52754, 1.552468,1.69869,1.831647,2.121587,2.387227, 2.640563

From the above data we construct the general progressive type-II censored data with standard notations: i = i-th failure observed, x_i = i-th failure observed time, R_i = number of withdrawals at i-th failureobserved, presented in Table 1. Here we have n=15,m=9, r=3.

Manufacturers also assume that the consumer satisfies the product if its life time is at least L which is the median of the posterior predictive distribution. The standard warranty under the FRW policy is set as 10th percentile of the posterior predictive distribution which is denoted by t_w. Suppose that the manufacturer wishes to set the percentage of benefit remains to be 0.8 (80%) under combined policy, then putting this value in the equation (21) we get the value ofA_2. The values of L and t_w are shown in Table 2 to Table 6 for different values of δ and υ.

Table 1. General progressive type-II censored data

i	1	2	3	4	5	6	7	8	9
x_i	-	-	-	1.183377	1.423381	1.552466	1.69869	1.831647	2.38722
R_i	-	-	-	2	2	0	0	1	1

Based on the above assumptions the optimal warranty length and maximum value of expected utility function(MU) under FRW, PRW and MIX(combined) policies are calculated and the results are shown in the Table 2 to Table 6.

Table 2. Values of L, t_w , W_1^*, W_2^* and MU under fixed value of δ and different values of υ.

δ	υ	Policy	L	t_w	W_1^*	W_2^*	MU
5	5	FRW	1.724	1.085	1.048	-	51.390025
		PRW			-	1.2549	55.89536
		MIX			1.032	1.0807	66.54556

Table 2. Values of L, t_w , W_1^*, W_2^* and MU under fixed value of δ and different values of v (continued)

δ	v	Policy	L	t_w	W_1^*	W_2^*	MU
5	10	FRW	1.49	1.062	1.029	-	51.71353
		PRW			-	1.1741	55.88600
		MIX			1.02	1.0519	66.69742
	15	FRW	1.369	1.049	1.02	-	51.93064
		PRW			-	1.1305	55.8753
		MIX			1.015	1.0386	66.79666
	20	FRW	1.296	1.04	1.015	-	52.0927
		PRW			-	1.1036	55.86541
		MIX			1.011	1.0296	66.875817
	25	FRW	1.247	1.034	1.012	-	52.2044
		PRW			-	1.0855	55.8547
		MIX			1.01	1.0253	66.925033

Table 3. Values of L, t_w , W_1^*, W_2^* and MU under fixed value of δ and different values of v.

δ	v	Policy	L	t_w	W_1^*	W_2^*	MU
10	5	FRW	2.222	1.127	1.094	-	50.96900
		PRW			-	1.4124	56.76578
		MIX			1.058	1.1445	66.34413
	10	FRW	1.794	1.092	1.054	-	51.31276
		PRW			-	1.2795	56.989761
		MIX			1.036	1.0902	66.51141
	15	FRW	1.586	1.072	1.037	-	51.57196
		PRW			-	1.2086	55.541074
		MIX			1.025	1.064	66.63287
	20	FRW	1.463	1.059	1.037	-	51.57196
		PRW			-	1.2086	54.541074
		MIX			1.019	1.0492	66.63287
	25	FRW	1.383	1.05	1.021	-	51.91555
		PRW			-	1.1358	53.05522
		MIX			1.015	1.0396	66.79258

Table 4. Values of L, t_w , W_1^*, W_2^* and MU under fixed value of δ and different values of v.

δ	v	Policy	L	t_w	W_1^*	W_2^*	MU
15	5	FRW	2.864	1.17	1.159	-	50.752035
		PRW			-	1.5867	57.680951
		MIX			1.091	1.2273	66.22812
	10	FRW	2.16	1.123	1.089	-	51.00722
		PRW			-	1.3956	57.691911
		MIX			1.055	1.1373	66.364997
	15	FRW	1.836	1.096	1.059	-	51.27107
		PRW			-	1.294	57.700561
		MIX			1.038	1.0956	66.492475
	20	FRW	1.651	1.079	1.042	-	51.48029
		PRW			-	1.2319	57.788618
		MIX			1.028	1.0719	66.592281
	25	FRW	1.533	1.067	1.033	-	51.645853
		PRW			-	1.1903	57.81576
		MIX			1.022	1.0573	66.6682

Table 5. Values of L, t_w , W_1^*, W_2^* and MU under fixed value of δ and different values of v.

δ	v	Policy	L	t_w	W_1^*	W_2^*	MU
20	5	FRW	3.691	1.215	1.245	-	50.72018
		PRW			-	1.7749	58.3387
		MIX			1.13	1.3298	66.181699
	10	FRW	2.601	1.154	1.133	-	51.703609
		PRW			-	1.5208	58.35777
		MIX			1.078	1.1947	66.27077
	15	FRW	2.125	1.12	1.086	-	51.04614
		PRW			-	1.3858	58.63134
		MIX			1.053	1.1328	66.38564
	20	FRW	1.864	1.099	1.061	-	51.23527
		PRW			-	1.3036	58.73138
		MIX			1.039	1.0989	66.47521
	25	FRW	1.699	1.084	1.047	-	51.41336
		PRW			-	1.2486	58.789772
		MIX			1.031	1.0785	66.56057

Table 6. Values of L, t_w , W_1^*, W_2^* and MU under fixed value of δ and different values of v.

δ	v	Policy	L	t_w	W_1^*	W_2^*	MU
25	5	FRW	4.757	1.262	1.355	-	50.86832
		PRW			-	1.9744	56.32803
		MIX			1.174	1.4519	66.19458
	10	FRW	3.132	1.187	1.188	-	50.870394
		PRW			-	1.6543	56.998832
		MIX			1.105	1.2633	66.204936
	15	FRW	2.461	1.145	1.119	-	50.87041
		PRW			-	1.4836	57.162748
		MIX			1.071	1.1771	66.29345
	20	FRW	2.103	1.119	1.084	-	51.047785
		PRW			-	1.3796	57.58422
		MIX			1.052	1.1303	66.38628
	25	FRW	1.883	1.1	1.063	-	51.241199
		PRW			-	1.3102	57.593349
		MIX			1.041	1.1024	66.48072

7. Simulation study

In this section we have carried out a simulation study considering the two values of the parameter of the Pareto life time model as θ =2 and 12 and keep other necessary values same as defined in the numerical example. Also simulation is done 1000 times and the average values of warranty length and maximum value of expected utility function are calculated along with their standard errors in case of all the three policies. All the calculations are done by preparing a computer program in 'Visual Basic' language. The results are shown in the Table 7 to Table 16.

Table 7 to Table 11 contain optimum warranty length and expected utility function with their standard errors for θ=2, n=20 and different values of prior parameter δ and v under FRW, PRW and combined policy and the Table 12 to Table 16 are for θ= 12.

Table 7. Values of W_1^*, W_2^*, MU, Std W_1^*, Std W_2^* and Std MU under the fixed value of δ and different values of v.

δ	v	Policy	W_1^*	W_2^*	MU	StdW_1^*	StdW_2^*	Std MU
5	5	FRW	1.2082	-	50.896268	0.08067	-	0.08704
		PRW	-	1.7578	57.429566	-	0.16433	0.69682
		MIX	1.0945	1.2388	66.26433	0.03401	0.08759	0.07806
	10	FRW	1.1234	-	50.900154	0.03196	-	0.13927
		PRW	-	1.50852	57.447571	-	0.10935	0.55773
		MIX	1.0571	1.1429	66.39094	0.0198	0.04915	0.11284

Table 7. Values of W_1^*, W_2^*, MU, Std W_1^*, Std W_2^* and Std MU under the fixed value of δ and different values of v (continued)

δ	v	Policy	W_1^*	W_2^*	MU	StdW_1^*	StdW_2^*	Std MU
5	15	FRW	1.1032	-	50.944721	0.00733	-	0.50111
		PRW	-	1.37591	57.556511	-	0.07988	0.45169
		MIX	1.0392	1.0987	66.5105	0.01329	0.03255	0.11668
	20	FRW	1.1	-	50.956233	0	-	1.03887
		PRW	-	1.29504	57.708193	-	0.06198	0.3752
		MIX	1.0294	1.0745	66.6075	0.00937	0.02323	0.11094
	25	FRW	1.1	-	50.976124	0	-	1.64837
		PRW	-	1.24111	57.747196	-	0.05014	0.32034
		MIX	1.0231	1.059	66.68464	0.00715	0.01759	0.10328

Table 8. Values of W1*, W2*, MU, Std W1*, Std W2* and Std MU under the fixed value of δ and different values of *v*.

δ	v	Policy	W_1^*	W_2^*	MU	StdW_1^*	StdW_2^*	Std MU
10	5	FRW	1.3056	-	50.79160	0.10454	-	0.17366
		PRW	-	1.95614	58.23363	-	0.17323	0.64203
		MIX	1.1337	1.3425	66.21463	0.03928	0.10621	0.02421
	10	FRW	1.1645	-	50.80184	0.05013	-	0.08198
		PRW	-	1.64149	58.91863	-	0.11588	0.53688
		MIX	1.0805	1.2016	66.28942	0.0235	0.05918	0.07448
	15	FRW	1.1145	-	50.88684	0.02175	-	0.10227
		PRW	-	1.47348	58.95057	-	0.0847	0.44607
		MIX	1.0547	1.1368	66.39713	0.01553	0.03845	0.09167
	20	FRW	1.1015	-	50.89226	0.00418	-	0.32332
		PRW	-	1.37106	58.97071	-	0.06568	0.37446
		MIX	1.0405	1.1018	66.49518	0.01115	0.02708	0.09434
	25	FRW	1.1	-	50.90296	0	-	0.6591
		PRW	-	1.30287	58.9961	-	0.05307	0.32171
		MIX	1.0316	1.0801	66.57804	0.00839	0.02054	0.09127

Table 9. Values of W_1^*, W_2^*, MU, Std W_1^*, Std W_2^* and Std MU under the fixed value of δ and different values of v.

δ	v	Policy	W_1^*	W_2^*	MU	$StdW_1^*$	$StdW_2^*$	Std MU
15	5	FRW	1.4288	-	50.08147	0.12795	-	0.31025
		PRW	-	2.16415	57.96311	-	0.18066	0.58591
		MIX	1.1779	1.4656	66.22633	0.04346	0.12397	0.04228
	10	FRW	1.2244	-	50.75749	0.06338	-	0.05318
		PRW	-	1.78127	58.54572	-	0.12146	0.51092
		MIX	1.1071	1.2703	66.22879	0.02652	0.06923	0.03882
	15	FRW	1.1424	-	50.83473	0.03518	-	0.0882
		PRW	-	1.57623	58.61791	-	0.08906	0.43241
		MIX	1.0727	1.1815	66.31057	0.01758	0.04429	0.0672
	20	FRW	1.1092	-	50.92093	0.01531	-	0.08312
		PRW	-	1.45103	58.97843	-	0.069	0.37278
		MIX	1.0533	1.1332	66.40187	0.01257	0.0311	0.07729
	25	FRW	1.1008	-	50.93843	0.0024	-	0.22862
		PRW	-	1.36773	58.99363	-	0.05577	0.31848
		MIX	1.0413	1.1039	66.48502	0.00952	0.0233	0.07888

Table 10. Values of W_1^*, W_2^*, MU, Std W_1^*, Std W_2^* and Std MU under the fixed value of δ and different values of v.

δ	v	Policy	W_1^*	W_2^*	MU	$StdW_1^*$	$StdW_2^*$	Std MU
20	5	FRW	1.5764	-	50.46131	0.15039	-	0.43086
		PRW	-	2.3801	59.12756	-	0.18743	0.52632
		MIX	1.2259	1.6071	66.28577	0.04629	0.14061	0.07783
	10	FRW	1.2987	-	50.81843	0.07584	-	0.11528
		PRW	-	1.92715	59.13768	-	0.1263	0.47799
		MIX	1.1368	1.3495	66.20509	0.02934	0.07916	0.01188
	15	FRW	1.1845	-	51.76130	0.04449	-	0.0486
		PRW	-	1.6836	59.1835	-	0.09294	0.41472
		MIX	1.0926	1.2324	66.25028	0.01971	0.05047	0.04428
	20	FRW	1.1291	-	51.86136	0.02661	-	0.08593
		PRW	-	1.53474	59.41127	-	0.07212	0.36209
		MIX	1.0677	1.169	66.3271	0.01405	0.03528	0.0601
	25	FRW	1.1061	-	51.93533	0.01132	-	0.0704
		PRW	-	1.43551	59.89447	-	0.05824	0.31493
		MIX	1.0524	1.131	66.40592	0.0108	0.02648	0.06668

Table 11. Values of W_1^*, W_2^*, MU, Std W_1^*, Std W_2^* and Std MU under the fixed value of δ and different values of υ.

δ	υ	Policy	W_1^*	W_2^*	MU	StdW_1^*	StdW_2^*	Std MU
25	5	FRW	1.7477	-	50.96146	0.17091	-	0.52032
		PRW	-	2.60355	57.22238	-	0.19356	0.47313
		MIX	1.2766	1.7662	66.38128	0.04818	0.15593	0.10332
	10	FRW	1.3859	-	50.97337	0.08852	-	0.19118
		PRW	-	2.07829	57.686805	-	0.13051	0.44795
		MIX	1.1697	1.44	66.2144	0.03150	0.08894	0.02539
	15	FRW	1.2359	-	51.749645	0.05172	-	0.04074
		PRW	-	1.79528	57.71085	-	0.09634	0.40015
		MIX	1.115	1.2903	66.21462	0.02158	0.05677	0.02251
25	20	FRW	1.161	-	51.78152	0.03365	-	0.05852
		PRW	-	1.62166	57.84023	-	0.07488	0.35456
		MIX	1.0839	1.2097	66.27042	0.01548	0.03947	0.04396
	25	FRW	1.1207	-	51.8916	0.0209	-	0.07557
		PRW	-	1.50593	57.27091	-	0.06053	0.31095
		MIX	1.0647	1.1612	66.3401	0.01184	0.02914	0.05447

Table 12. Values of W_1^*, W_2^*, MU, Std W_1^*, Std W_2^* and Std MU under the fixed value of δ and different values of υ.

δ	υ	Policy	W_1^*	W_2^*	MU	StdW_1^*	StdW_2^*	Std MU
5	5	FRW	1.1	-	48.292679	0	-	0.96325
		PRW	-	1.18799	54.397966	-	0.0196	0.13137
		MIX	1.0196	1.0506	66.71836	0.00284	0.00683	0.04452
	10	FRW	1.1	-	43.935445	0	-	1.85842
		PRW	-	1.12884	54.496211	-	0.01326	0.09547
		MIX	1.0127	1.0335	66.84496	0.00168	0.00434	0.03613
	15	FRW	1.1	-	38.269195	0	-	2.85193
		PRW	-	1.09691	54.766834	-	0.00984	0.07113
		MIX	1.0102	1.0258	66.9181	0.0004	0.00199	0.02375
	20	FRW	1.1	-	31.63560	0	-	3.91434
		PRW	-	1.07715	55.826854	-	0.00774	0.05596
		MIX	1.01	1.0225	66.94067	0	0.00157	0.00701
	25	FRW	1.1	-	24.25081	0	-	4.99327
		PRW	-	1.06385	55.91425	-	0.00635	0.0477
		MIX	1.01	1.0206	66.9943	0	0.0008	0.03468

Table 13. Values of W_1^*, W_2^*, MU, Std W_1^*, Std W_2^* and Std MU under the fixed value of δ and different values of v.

δ	v	Policy	W_1^*	W_2^*	MU	$StdW_1^*$	$StdW_2^*$	StdMU
10	5	FRW	1.1	-	46.67993	0	-	0.09053
		PRW	-	1.33631	55.016151	-	0.02251	0.13607
		MIX	1.041	1.103	66.47006	0.004	0.00979	0.03249
	10	FRW	1.1	-	46.7765	0	-	0.35903
		PRW	-	1.22866	55.0231	-	0.01505	0.09677
		MIX	1.0256	1.0657	66.6318	0.00254	0.0058	0.0301
	15	FRW	1.1	-	47.791352	0	-	0.66231
		PRW	-	1.17101	55.027895	-	0.0111	0.07579
		MIX	1.0184	1.0476	66.74239	0.00162	0.00388	0.02642
	20	FRW	1.1	-	49.157105	0	-	0.99702
		PRW	-	1.13551	55.047063	-	0.00874	0.05969
		MIX	1.0141	1.037	66.82067	0.0013	0.00319	0.02292
	25	FRW	1.1	-	50.995223	0	-	1.35378
		PRW	-	1.11163	55.868342	-	0.00716	0.05032
		MIX	1.0114	1.0301	66.87896	0.00102	0.00239	0.0205

Table 14. Values of W_1^*, W_2^*, MU, Std W_1^*, Std W_2^* and Std MU under the fixed value of δ and different values of v.

δ	v	Policy	W_1^*	W_2^*	MU	$StdW_1^*$	$StdW_2^*$	Std MU
15	5	FRW	1.1215	-	50.85341	0.00999	-	0.03278
		PRW	-	1.50306	55.26216	-	0.02495	0.12834
		MIX	1.0697	1.174	66.30151	0.00494	0.01247	0.02018
	10	FRW	1.1	-	50.92361	0	-	0.05776
		PRW	-	1.33988	55.36127	-	0.01656	0.1012
		MIX	1.0426	1.107	66.46055	0.00284	0.00717	0.02323
	15	FRW	1.1	-	50.9534	0	-	0.18815
		PRW	-	1.2531	55.82827	-	0.01219	0.07536
		MIX	1.0298	1.0754	66.58526	0.00194	0.00476	0.0226
	20	FRW	1.1	-	50.96380	0	-	0.33868
		PRW	-	1.19987	56.48382	-	0.00951	0.06821
		MIX	1.0225	1.0577	66.6798	0.00157	0.00363	0.02044
	25	FRW	1.1	-	50.9784	0	-	0.50654
		PRW	-	1.16421	57.24399	-	0.00775	0.05431
		MIX	1.018	1.0466	66.75227	0.00118	0.00284	0.0186

Table 15. Values of W_1^*, W_2^*, MU, Std W_1^*, Std W_2^* and Std MU under the fixed value of δ and different values of v.

δ	v	Policy	W_1^*	W_2^*	MU	StdW_1^*	StdW_2^*	Std MU
20	5	FRW	1.1956	-	50.71778	0.01334	-	0.1007
		PRW	-	1.685	55.14570	-	0.02687	0.12381
		MIX	1.1052	1.2644	66.21039	0.0061	0.01571	0.00842
	10	FRW	1.1083	-	50.91495	0.00625	-	0.02883
		PRW	-	1.46103	56.03845	-	0.01791	0.09574
		MIX	1.0634	1.1582	66.33262	0.00364	0.00892	0.01655
	15	FRW	1.1	-	50.94441	0	-	0.0377
		PRW	-	1.34199	56.37459	-	0.01311	0.07723
		MIX	1.0434	1.1091	66.45519	0.00233	0.00573	0.01804
	20	FRW	1.1	-	50.95761	0	-	0.12444
		PRW	-	1.26932	56.93165	-	0.0102	0.06599
		MIX	1.0326	1.0823	66.55649	0.00169	0.0042	0.0176
	25	FRW	1.1	-	50.97318	0	-	0.021029
		PRW	-	1.22081	57.62269	-	0.00829	0.05035
		MIX	1.0256	1.0655	66.63819	0.00143	0.00332	0.01675

Table 16. Values of W_1^*, W_2^*, MU, Std W_1^*, Std W_2^* and Std MU under the fixed value of δ and different values of v.

δ	v	Policy	W_1^*	W_2^*	MU	StdW_1^*	StdW_2^*	Std MU
25	5	FRW	1.3001	-	50.76500	0.01605	-	0.02121
		PRW	-	1.86147	56.79719	-	0.02835	0.11479
		MIX	1.1467	1.375	66.18847	0.00681	0.01866	0.00163
	10	FRW	1.1572	-	50.78639	0.00852	-	0.01714
		PRW	-	1.59077	56.80068	-	0.01905	0.09378
		MIX	1.088	1.2198	66.24706	0.004	0.01041	0.01015
	15	FRW	1.1024	-	50.95037	0.00332	-	0.0261
		PRW	-	1.43706	56.90997	-	0.01395	0.07835
		MIX	1.0598	1.1492	66.35207	0.00668	0.00679	0.01419
25	20	FRW	1.1	-	50.95766	0	-	0.02949
		PRW	-	1.34338	56.97975	-	0.01082	0.06671
		MIX	1.0442	1.1108	66.45164	0.00209	0.00492	0.01479
	25	FRW	1.1	-	50.98060	0	-	0.08617
		PRW	-	1.28092	57.00520	-	0.00881	0.05289
		MIX	1.0345	1.087	66.53715	0.00157	0.00379	0.01472

8. Conclusion

In today's competing market product warranty plays an increasingly significant role in both consumer and commercial transactions. For manufacturer it is important to decide the appropriate warranty length and appropriate warranty policy so that he may increase the demand of his product and hence makes more profit. We have provided an approach to the manufacturers to determine optimal warranty length and warranty policy based on the life time data obtained by conducting life testing experiment using general progressive type-II censoring scheme for the product having Pareto life time distribution. Based on the life data obtained through such a life test a Bayesian predictive distribution is derived to determine the maximum value of utility function and hence an appropriate warranty policy is decided.

Based on the output of the example considered in Section 6 for various combinations of the values of the prior parameters we observed from the Table 2 to Table6 that, for the given data combined policy gives maximum utility followed by PRW and then by FRW policy for any choice of prior parametersδ and v. We have also examined the effect of change in the value of one prior parameter when the value of other parameter kept fixed. For any fixed value of prior parameter δ as v increases, maximum utility decreases in all the three types of policies and for keeping v fixed, as δ increases, maximum utility more or less remains stable. Thus maximum utility has more effect of prior parameter v compare to the parameter δ.

A simulation study carried out in Section 7 shows a very general effect of the prior parameters on different types of warranty policies. The results are shown in the Table 7 to Table 16. From the Table 7 to Table 16 we observed that combined policy gives maximum utility followed by PRW and then by FRW policy for any choice of prior parameters. For any fixed value of prior parameter δ as v increases, maximum utility decreases in all the three types of policies and for fixed v as δ increases, the value of maximum utility fluctuates.

Thus for the product having decreasing failure rate and Pareto life time distribution we suggest to utilize a mixed warranty policy which is a combination of PRW and PRW policies with any values of the prior parameters utilized in the model.

This paper becomes useful to determine optimal warranty of those products which have only Pareto life time distribution which is the limitation of the paper. Many papers are also available to determine the optimal warranty for the product having different life time distributions possessing constant, increasing or decreasing failure rates. The drawback of such papers is to first know the actual life time distribution of the product and then one can apply the appropriate method to decide the optimal warranty. To overcome such a difficulty we are preparing a paper based on the very general life time distribution which may be useful to the product having increasing, constant or decreasing failure rate based on Weibull life time model. But there are many products which possess the failure rate initially decreasing, then after becomes constant for certain period of time and then gradually increases with time. For such types of products thesekind of work may not be useful. One has to develop a very general model possessing a bath tub failure rate distribution to determine optimal warranty of such types of product.Very few papers are available in the literature which might be useful to determine the optimum price as well as the optimal warranty of the product; one can do also such kind of work in this direction.

Acknowledgement: The Authors are grateful to the Editor- in -Chief and to the referee for their detailed comments and suggestions.

References:

Aggarwala, R., & Childs, A. (2000). Conditional inference for the parameters of Pareto distributions when observed samples are progressively censored. In N. Balakrishnan, V. B. Melas, S. Ermakov (Eds.), *Advances in Stochastic Simulation Methods* (pp. 293-302). Boston, MA: Birkhäuser.

Balakrishnan, N., Rad, A. H., & Arghami, N. R. (2007). Testing exponentiality based on Kullback-Leibler information with progressively Type-II censored data. *IEEE Transactions on Reliability*, *56*(2), 301-307.

Cohen, A. C. (1963). Progressively censored samples in life testing. *Technometrics*, *5*(3), 327-339.

Gajjar, K. A., & Patel, M. N. (2008). Estimation for a mixture of exponential distributions based on progressively type-II censored sample. *International journal of agricultural and statistical sciences*, *4*(1), 169-176.

Gouno, E., Sen, A., & Balakrishnan, N. (2004). Optimal step-stress test under progressive Type-I censoring. *IEEE Transactions on Reliability*, *53*(3), 388-393.

Hossain, A. M., & Zimmer, W. J. (2000). Comparisons of methods of estimation for a Pareto distribution of the first kind. *Communications in Statistics-Theory and Methods*, *29*(4), 859-878.

Jevakumar, K., & Robert, T. P. (2010). Joint determination of price, warranty length and production quantity for new products under free renewal warranty policy. In *Proceeding of the 4 th International Quality Conference*.

Djamaludin, I., Murthy, D. N. P., & Blischke, W. R. (1996). Bibliography on warranties. In: W. R. Blischke, D. N. P. Murthy (Eds.), *Product warranty handbook*, (pp. 839-916). New York.

Lawless, J. F. (2011). *Statistical models and methods for lifetime data*. Hoboken, New Jersey: John Wiley & Sons.

Mann, N. R. (1971). Best linear invariant estimation for Weibull parameters under progressive censoring. *Technometrics*, *13*(3), 521-533.

Nadi, A. A., & Gildeh, B. S. (2016). Estimating the lifetime performance index of Products for Two-Parameter Exponential Distribution with the Progressive First-Failure Censored Sample. *International Journal for Quality Research*, *10*(2), 389-406.

Patel, D. T., & Patel, M. N., (2017). A Bayesian approach to optimal warranty length for a Kumaraswamy life time distributed product with general progressive censoring scheme. *International Journal of Statistics and Economics*, *18*(2), 93-106.

Patel, N. W., & Patel, M. N. (2007). Progressively type-II censored sample from geometric life time model. *Journal of Probability and Statistical Science*, *5*(1), 81-95.

Podder, C. K., Roy, M. K., & Karim, A. (2004). Minimax estimation of the parameter of the Pareto distribution under Quadratic and MLINEX loss functions. *Pakistan journal of statistics-all series*, *20*(1), 137-150.

Scitovszky, T. (1944). Some consequences of the habit of judging quality by price. *The Review of Economic Studies*, *12*(2), 100-105.

Shah, J. B., & Patel, M. N. (2007). Estimator of parameter in a pareto distribution from multiply type II censored data. *International journal of agricultural and statistical sciences*, *3*(2), 581-588.

Singpurwalla, N. D., & Wilson, S. P. (1998). Failure models indexed by two scales. *Advances in Applied Probability*, *30*(4), 1058-1072.

Viveros, R., & Balakrishnan, N. (1994). Interval estimation of parameters of life from progressively censored data. *Technometrics*, *36*(1), 84-91.

Wu, C. C., Lin, P. C., & Chou, C. Y. (2006a). Determination of price and warranty length for a normal lifetime distributed product. *International Journal of Production Economics*, *102*(1), 95-107.

Wu, S. J., Chen, D. H., & Chen, S. T. (2006b). Bayesian inference for Rayleigh distribution under progressive censored sample. *Applied Stochastic Models in Business and Industry*, *22*(3), 269-279.

Wu, S. J., & Huang, S. R. (2010). Optimal warranty length for a Rayleigh distributed product with progressive censoring. *IEEE Transactions on Reliability*, *59*(4), 661-666.

Divya T. Patel
SLU Arts and H & P Thakor Commerce College,
Ellisbridge,
Ahmedabad,
India
divyapatel.1985@yahoo.com

Manharlal N. Patel
Gujarat Uniesity, Schhol of Sciences, Depatment o Statistics,
Ahmedabad,
India
mnpatel.statgmail.com