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SIX SIGMA-BASED RANGE AND STANDARD DEVIATION CHARTS FOR CONTINUOUS QUALITY IMPROVEMENT

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Abstract: *Control charts for range (R-chart) and standard deviation (S-chart) are commonly used charts along with X-bar control chart for variables. One of the major challenges in these charts is the use of appropriate estimators of the unknown population parameters involved in the control limits. In this paper, first a detailed review on the classical charts for dispersion, R-chart and S-chart, is presented and then by applying the concept of Six Sigma quality characteristics, new set of Six Sigma-based R- and S-charts are developed. The motivation of the proposed approach is due to the fact that "specifications refer to the deviations that are permissible from the target and the units produced from a well-behaved process will match the target and standard deviation associated with its specifications". Accordingly, unlike the traditional charts, in this new approach, the unknown population standard deviation related to range and standard deviation are derived from specification using the perspective of Six Sigma. Procedures for obtaining control limits of the proposed Six Sigma-based R-and S-chart are given. The average run length values for the proposed new charts are also obtained for different in-control and out-of control shift values. It is discussed that due to various reasons a process may maintain certain sigma quality level at a point of time that may be in terms of number of sigma. Since the goal of Six Sigma is of 3.4 defects per million opportunities, it is recommended to keep monitoring the process in terms of sigma quality level by using the improved control limits for the purpose of continuous quality improvement every time till the goal is achieved. The proposed charts are illustrated with appropriate numerical examples for better understanding.*

Keywords: *average run length, defects per million opportunities, Six Sigma-based R- and S-charts, Six Sigma quality, traditional R- and S-charts*

1. Introduction

In today's customer-oriented competitive marketplace, manufacturing firms and service providers are expected to meet the changing needs of customers continuously. In fact, the presence of huge product variants and the growing needs of customers force organizations to come out with robust, high quality and cost-effective products. In order to improve quality of processes and

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products, and hence to achieve business excellence, organizations adopt Total Quality Management (TQM) methodology. Since contribution of quality is the key to this achievement, the focus of any quality improvement tools and techniques is expected to be about variability reduction, cost savings and hence meeting the goals successfully (Andersson et al., 2006). Asilahijani et al. (2010) pointed out that higher variation in a quality characteristic always results in undesirable outcomes with regard to cost of rework or scrap, customer dissatisfaction and poor functional activities. Therefore, variance reduction in any key product/process quality characteristic plays a major role in quality improvement programs. Six Sigma quality program is one of the means to achieve the benefits of TQM mainly through reduced variability and increased output levels. The purpose of Six Sigma quality program is to achieve organizational goals through systematic application of statistics-based tools and techniques.

Statistical Process Control (SPC) is an effective tool for achieving quality improvement. Controlling and monitoring of processes/products statistically are the key aspects of any SPC activity. Statistical Quality Control (SQC) charts - Shewhart control charts for attributes and variables- are commonly used for this purpose. In fact, the use and nature of control charts are different for variables and attributes. While X-bar Chart is useful in detecting any shift in the mean (an out-of-control point), it is always preferable to check whether there is any out-of-control state in the dispersion as well. R- and S-charts are important statistical process control (SPC) charts that can help to find if there is any high level of dispersion in the data that needs attention. This is essential because every sub-sample contains n observations and there is a possibility that the difference between the maximum and minimum observations is high or the spread of the observations is high indicating an out-of-control situation. Therefore, while R-Chart is useful in checking whether a process is under statistical control with regard to range, S-chart is useful in knowing whether the process is under control with regard to spread of the data. It may be noted that range and spread give an indication over the quantum of variability in the process data.

Though there exist a number of control charts for different process situations as proposed by researchers and practitioners, developments in use and application of control charts keep evolving over years mainly due to industrial automation and requirements in information systems and technology. Most of these developments are based on the use of appropriate estimators (such as robust and efficient) for unknown parameters involved in the construction of control charts. Also, high quality processes, in fact, warrants more sophisticated and but still efficient control charts and tools for process monitoring (Box and Narasimhan, 2010). This poses a major challenge to Six Sigma quality practitioners to decide on the type of control chart that is more appropriate for the process/product being monitored. Accordingly, effective use of the existing control chart(s) or the introduction of new chart(s) is always appreciated by Six Sigma practitioners who often deal with high quality processes and are in search of easy-to- use and advanced quality control charts. According to Hsu et al. (2009), application of traditional control charting and reliability methods may not yield significant results when they are used for a highly reliable process and products. Therefore, there exists a need for the development of sophisticated control charts that can help to continually improve and ensure highly reliable and quality processes and products. It may be noted that, variability reduction being the key focus of Six Sigma quality program, maintaining a process close to the target by means of reducing process variation is an important task of any organization aiming for high quality processes and products.

A controlled process is naturally meeting the set specifications (target and specification limits) resulting in minimum loss to both producer and consumer. The distribution of units of a well behaved process always matches the target of the specification limits and hence the standard deviation. Keeping this in mind, a new method of fixing a Six Sigma Quality-based specification for quality characteristics was suggested by Ravichandran (2006). In this paper, we propose new control charts for dispersion (Six Sigma-based R- and S-charts) by estimating the standard deviation using the process/product specification from the perspective of Six Sigma quality of 3.4 defects per million opportunities (DPMO). The process under this quality level may go out-of-control (either below lower control limit (LCL) or above upper control limit (UCL)) with probability 0.0000068 only due to higher level of range/spread among the observations, whereas the process under traditional R- and S-charts may go out-of-control with probability 0.0027 leading to 1349.97 DPMO. The proposed charts also overcome the problems faced in the application of traditional charts where estimation of unknown parameters for use in the development control limits is a major concern. In addition, we show how to determine the current sigma level of the process being monitored so as to decide if further (and how much) improvement is required.

2. Literature Review

As discussed in the introduction section, there has been a growing interest in developing control charts and their applications depending upon the nature of processes and products. There are many recent studies made on such advanced control charts related to different areas of interest. In their work, Gadre and Rattihalli (2004) combined Shewart X-bar chart and a *group runs* chart to propose a synthetic control chart called group runs control chart that can detect small shifts in the process mean. Hsu et al. (2009) considered a two-stage t-chart controlling procedure for reliability monitoring and performance measuring of an exponential failure process with high reliability. Hassan et al. (2010) examined the effective use of SPC charting methodology for controlling market risks in case of systematic trading and investment. Ryan and Woodall (2010) studied the efficiency of various cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control charts when data from a process with Poisson count are used with varying sample sizes.

Ryu et al. (2010), observed the necessacity of optimally designing a CUSUM chart, particularly when the size of the mean shift is assumed unknown. Reynolds and Lou (2010) pointed out that the traditional X-bar chart is not effective in detecting small shifts in the mean of a normal process. It is further observed that though CUSUM charts and EWMA charts are very effective for detecting small shifts in the process mean, they are not able detect the presence of large shifts. According to Reynolds and Lou (2010), since size of the shift in the process mean is usually unknown, they proposed a chart that it is able to effectively detect a wide range of shift sizes. Zhu and Lin (2010), worked on the problem of monitoring the slopes of linear profiles using the Shewhart-type T^2 chart. Schoonhoven et al. (2011) studied control charts using different estimators of standard deviation then assessed the impact of those estimators on implementation. Lee (2011) proposed adaptive R-charts by extending the features of adaptive control charts to the traditional R-chart with variable parameters. It is noticed that such chart can improve the efficiency of signaling high process variation. Recently, Jones et al. (2014) reviewed the aspects of collection and analysis of data for use in process improvement and control charting. While presenting a good review of a number of papers published in this area, Jones et al.,

(2014) discussed about the importance of using good set of data for estimation of the unknown parameters that are used in control charts.

It is interesting to note that most of these charts face problems in estimating the unknown parameters of interest since parametric estimation significantly influences the efficiency of the control chart in detecting signals. In case of the commonly used X-bar chart, the average range is used, at times, in the construction control limits when population standard deviation is unknown. Schoonhoven et al. (2009) considered five unbiased estimators (pooled sample standard deviation, mean sample standard deviation, mean sample range, Gini's mean sample differences, mean sample inter-quartile range) of population standard deviation and studied various design schemes when there is a shift in the process mean from the population mean. Two of these estimators involve average range and average standard deviation as well. Clearly, the problem still remains open as there can be better estimators for the unknown population standard deviation. In fact this aspect has motivated the author for the present work.

Radhakrishnan and Balamurugan, (2011a, 2011b) studied the case of constructing control charts separately for range and standard deviation with the Six Sigma initiatives taking into account the standard deviation is determined in terms of known process tolerance and process capability index. However, in SPC applications, it is advisable to determine the capability of the process after ensuring that the process is under control. Using the concept of Six Sigma Quality characteristics, Ravichandran (2016) developed a Six Sigma control chart for variable (X-bar chart) that can be applied by organizations to achieve higher quality levels. In our work, we start with the following information related to specification limits and control limits. While specifications refer to the deviations that are permissible from the target, or the end product that is aimed, control limits are based upon past performance. In fact, the limits of variation arising from a process are referred to as control limits when the process is under statistical control.

The remainder of the paper is organized as follows. Section 3 discusses the aspects of Phase I and Phase II control charts. In Section 4, the features of traditional R- and S control charts are reviewed and the construction of control limits is presented. The proposed new Six Sigma-based R- and S-control charts are presented in Section 5. A study on the performance measures of the proposed R- and S-charts is made in Section 6. In Section 7 numerical examples are given to illustrate the working of the proposed Six Sigma-based R- and S-charts. The determination of average rung length for these specific examples is also given. The summary and conclusions are presented in Section 8.

3. Phase I and Phase II control charts

In practice, once a control chart is developed, then it has to be used for monitoring the process of interest. However, the process parameters which are essential in the construction of control chart are often unknown to the experimenter. There are two phases (Phase I and Phase II) in the development and application of control charts. In Phase I, a common statistical procedure is followed to estimate the parameters from samples taken when the process is in control or assumed to be in control (for more readers are referred to Woodall and Montgomery 1999; Vining, 2009; Schoonhoven et al., 2011). In Phase II of the control charting process, the control chart developed by the estimated parameters in Phase I is applied for monitoring the process.

Chakraborti et al. (2008) presented an overview and some results related to the development of Phase I charts. According to

Chakraborti et al. (2008), a process is said to be in control with no presence of any assignable cause(s), if the quality characteristic of interest being monitored is close enough to the set target. In the event that any significant deviation is noticed in Phase I, then it is often suggested to keep updating the control limits by eliminating such out-of control incidences so that a reasonable set of control limits are used in Phase II for monitoring the process. Jensen et al. (2006) suggested for the use of robust estimators in Phase I and at the same time they pointed out the importance of assessing the effectiveness of such estimators in Phase II applications. Schoonhoven et al. (2011) presented a study on design and analysis of control charts for standard deviation in which several estimators of standard deviation are used in Phase I and then assessed their impact during implementation in Phase II. Some of these estimators include, the commonly used average sample standard deviation and average sample range. The performances of charts with respective estimators are numerically evaluated. As discussed in the review of literature, use of appropriate (robust and efficient) estimator for unknown population standard deviation still remains a challenge to researchers and practitioners.

In this paper, we propose to use the standard deviation, say σ_X, estimated from the process/product specification of a quality characteristic, X, from the perspective of Six Sigma quality in the construction of Six Sigma-based R- and S-charts in Phase I. As discussed by Schoonhoven et al. (2011), while applying these charts in Phase II, if the process standard deviation, say $\hat{\sigma}$, is equal to σ_X, we call the process is in control, otherwise, the process is assumed to have shifted and in this case we have $\hat{\sigma} = \gamma\sigma_X, \gamma > 1$. If $\gamma = 1$, we get an in-control process standard deviation $\hat{\sigma} = \sigma_X$. It may be noted that since our estimation is based on Six Sigma quality, we allow a shift of $\gamma = 1.5$ times of standard deviation (for both R- and S-charts) as this process still ensures just 3.4 DPMO. This aspect is considered while computing the performance measure, Average Run Length (ARL).

4. Traditional range and standard deviation charts

4.1. Traditional range chart (R-chart)

The Shewhart-type control chart for range (Shewhart, 1931) can be found in most of the statistics text books. For more on this chart readers are referred to Ravichandran (2010) and Montgomery and Runger (2005). Let R be the random variable representing range and let $R_1, R_2, \cdots\cdots, R_k$ be the range values computed from k sub-samples, each of size n, then the traditional three-sigma control limits for range R is given by:

$$\bar{R} \pm 3 d_3 \bar{R} / d_2 \tag{1}$$

Where

$$\bar{R} = \frac{1}{k}\sum_{i=1}^{k} R_i, \quad R_i = \max \; x_{ij} - \min \; x_{ij},$$

$$i = 1,2,\cdots\cdots,k, \quad j = 1,2,\cdots\cdots n$$

x_{ij} is the j^{th} observation of i^{th} sample of some measurable characteristic, say X. Here, d_2 and d_3 are the constants for a given sub-sample size n tabulated in most texts books on statistics. It may be noted that the standard deviation of the range R is originally given as $\sigma_R = \sigma_X d_3$ where σ_X is the population standard deviation of the random variable X. Since σ_X is unknown, it is replaced by an estimate $\bar{R} / d_2$. Hence, the distribution of R (Gumbel, 1947) has mean $\bar{R}$ and approximated standard deviation $\hat{\sigma}_R = d_3 \bar{R} / d_2$, using central limit theorem, we have

$$P[\overline{R} - 3\hat{\sigma}_R \le R \le \overline{R} + 3\hat{\sigma}_R] = 1 - 0.0027 = 0.9973 \quad (2)$$

Therefore, the range R_i of i^{th} sample is said to be in control if

$$\overline{R} - 3\hat{\sigma}_R \le R_i \le \overline{R} + 3\hat{\sigma}_R \quad (3)$$

and is out of control otherwise.

4.2. Traditional standard deviation chart (S-chart)

In addition to R-chart, Shewhart (1931) stressed the importance of analyzing the spread of the data using the control chart for sample standard deviations, called S-chart. This is due to the reason that an in-control state in an S-chart shows the stability of the process. If x_{ij} is the j^{th} observation of i^{th} sample of the measurable characteristic, say X, then we know that an unbiased estimator of σ_X^2 is the average of the k sub-sample variances (each subsample is of size n) given by,

$$\overline{S}^2 = \frac{1}{k}\sum_{i=1}^{k} S_i^2$$

where:

$$S_i^2 = \frac{1}{n-1}\sum_{j=1}^{n}(x_{ij} - \overline{X}_i)^2, \quad i = 1, 2, \cdots\cdots, k$$

are the k sub-sample variances. Here,

$$S_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(x_{ij} - \overline{X}_i)^2}, \quad i = 1, 2, \cdots\cdots, k$$

are the sub-sample standard deviations. However, recall that the average sample standard deviation given by $\overline{S} = \frac{1}{k}\sum_{i=1}^{k} S_i$ is not an unbiased estimator of σ_X.

If the underlying distribution is normal, then $\overline{S}$ actually estimates $c_4\sigma_X$, where c_4 is a constant that depends on the size n of the sub sample. The constant c_4 is tabulated in most text books on statistics and may be calculated as given below.

$$c_4 = \sqrt{2/(n-1)}\,\Gamma(n/2)/\Gamma[(n-1)/2]$$

Note that $\Gamma m = (m-1)!$ and $\Gamma(1/2) = \sqrt{\pi}$.

Therefore, we have,

$$E(\overline{S}) = c_4\sigma_X$$

and

$$V(\overline{S}) = \sigma_S^2 = \sigma_X^2(1 - c_4^2)$$

Therefore, $\sigma_S = \sigma_X\sqrt{1 - c_4^2}$ gives the sample standard deviation of S. If the population standard deviation σ_X is known, then the 3-sigma control limits for standard deviation are given as:

$$c_4\sigma_X \pm 3\sigma_X\sqrt{1 - c_4^2} = c_4\sigma_X + 3\sigma_S$$

However, if the population standard deviation σ_X is not known then we can use $\overline{S}/c_4$ as an unbiased estimator of σ_X and hence the 3-sigma control limits become:

$$\overline{S} \pm 3\frac{\overline{S}}{c_4}\sqrt{1 - c_4^2} \quad (4)$$

Hence, the distribution of S has mean $\overline{S}$ and approximated standard deviation $\hat{\sigma}_S = (\overline{S}/c_4)\sqrt{1 - c_4^2}$. Now, using central limit theorem, we have:

$$P[\overline{S} - 3\hat{\sigma}_S \le S \le \overline{S} + 3\hat{\sigma}_S] = 1 - 0.0027 = 0.9973 \quad (5)$$

Therefore, the standard deviation S_i of i^{th} sample is said to be in control if:

$$\bar{S} - 3\hat{\sigma}_S \le S_i \le \bar{S} + 3\hat{\sigma}_S \quad (6)$$

and is out-of-control otherwise.

In fact, in addition to the estimators $\bar{R}/d_2$ and $\bar{S}/c_4$ of the unknown population standard deviation σ_X used in the traditional control charts for dispersion, Schoonhoven et al. (2009, 2011) proposed to apply a number of estimators such as pooled sample standard deviation, trimmed mean of sample standard deviations, mean of sample standard deviations after trimming the sample observations, mean of sample inter-quartile ranges, mean of the sample Gini's mean differences, mean of sample averages of absolute deviation from the median, mean of the sample medians of the absolute deviation from the median, mean of sample medians of the absolute deviation from the mean and Tatum's robust estimator. Whatever be the estimator used, it may be noted from (2) and (5) that the process under traditional R- and S-charts may go out-of-control with probability 0.0027 leading to 1349.97 DPMO that may not be often acceptable to the quality practitioners whose aim is to have zero defect process. This is the prime focus of our research in which we establish that the proposed charts meet the Six Sigma quality requirement of just 3.4 DPMO.

5. Six Sigma-based R- and S-charts

5.1. Six Sigma-based R-chart

Given the specification limits for the measurable characteristics, say X, we have lower specification limit (LSL) and upper specification limit (USL). It is known that given X that follows normal process with mean $T = \mu$ and variance σ^2, the specification of X is usually given in the form $T \pm K\sigma_X$, where T is the target or population mean, K is a positive constant and σ_X is the population standard deviation. Now, we can estimate the unknown population standard deviation as $\sigma_X = d/K$ (refer to Ravichandran, 2006), where $d = K\sigma_X$ gives half of the process spread USL – LSL. Since, range and standard deviation are closely related (Schwarz, 2006), the standard deviation of R for Six Sigma quality becomes.

$$\hat{\sigma}_{RS} = \sigma_X d_3 = (d/K)d_3 \quad (7)$$

For a typical Six Sigma Quality process we have $K = 6$ and hence $\sigma_X = d/6$ Accordingly, since the distribution of R (Gumbel, 1947) has mean $\bar{R}$ and standard deviation $\hat{\sigma}_{RS} = (d/6)d_3$. Now, by central limit theorem we have $Z = (R - \bar{R})/\hat{\sigma}_{RS}$ which is the standardized normal variate and hence we have

$$P[|Z| \le Z_\alpha(K_R)] = P[\bar{R} - Z_\alpha(K_R)\hat{\sigma}_{RS} \le R \le \bar{R} + Z_\alpha(K_R)\hat{\sigma}_{RS}]$$

For a Six Sigma quality with centered process, we have $\alpha = 2x10^{-9}$, that is $P(|Z| \le 6) = 1 - 2x10^{-9}$ with $z_\alpha(K_R) = 6$ which implies:

$$P[\bar{R} - 6\hat{\sigma}_{RS} \le R \le \bar{R} + 6\hat{\sigma}_{RS}] = 1 - 2x10^{-9}$$

$$\Rightarrow P[R \le \bar{R} - 6\hat{\sigma}_{RS}] = P[R \ge \bar{R} + 6\hat{\sigma}_{RS}] = 1x10^{-9}$$

Therefore, the Six Sigma-based control limits for range can be given as:

$$\bar{R} \pm z_{\alpha_R}(K_R)\hat{\sigma}_{RS} \quad (8)$$

Where $z_{\alpha_R}(K_R)$ is the constant such that:

$$P[-z_\alpha(K_R) \le Z \le +z_\alpha(K_R)] = 1-\alpha_{K_R} \tag{9}$$

$$\alpha_{K_R} = (2)(DPMO \text{ corresponding to } K_R)10^{-6}$$

Here, K_S is the current *quality level* with an allowable shift of 1.5 times of standard deviation at which the process is needed to be controlled. For example, if $K_S = 6$, then with shift, we have $DPMO = 3.4$ either on left tail or on right tail. Therefore, $\alpha_{K_S} = 6.8x10^{-6}$ which implies $z_\alpha(K_S) = 4.5$. Therefore, the control limits for Six Sigma-based S-chart become:

$$\overline{R} \pm 4.5\hat{\sigma}_{RS} \tag{10}$$

and accordingly we have:

$$P[\overline{R} - 4.5\hat{\sigma}_{RS} \le R \le \overline{R} + 4.5\hat{\sigma}_{RS}] = 1 - 6.8x10^{-6} = 0.9999932 \tag{11}$$

Therefore, the range R_i of i^{th} sample is said to be in control if

$$\overline{R} - 4.5\hat{\sigma}_{RS} \le R_i \le \overline{R} + 4.5\hat{\sigma}_{RS} \tag{12}$$

and is out- of- control otherwise.

5.2. Six Sigma-based S-chart

In case of traditional S-chart, we have $\sigma_S = \sigma_X\sqrt{1-c_4^2}$.

As discussed above for R-chart, for a typical Six Sigma Quality process we have the standard deviation as:

$$\hat{\sigma}_{SS} = (d/6)\sqrt{1-c_4^2} \tag{13}$$

and hence by central limit theorem we have $Z = (S-\overline{S})/\hat{\sigma}_{SS}$ which is the standardized normal variate and hence we have:

$$P[|Z| \le Z_\alpha(K_S)] = P[\overline{S} - Z_\sigma(K_S)\hat{\sigma}_{SS} \le S \le \overline{S} + Z_\alpha(K_S)\hat{\sigma}_{SS}]$$

For a Six Sigma quality with centered process, we have $\alpha = 2x10^{-9}$, that is $P(|Z| \le 6) = 1 - 2x10^{-9}$ with $z_{\alpha_S}(K_S) = 6$ which implies:

$$P[\overline{S} - 6\hat{\sigma}_{SS} \le S \le \overline{S} + 6\hat{\sigma}_{SS}] = 1 - 2x10^{-9}$$

$$\Rightarrow P[S \le \overline{S} - 6\hat{\sigma}_{SS}] = P[S \ge \overline{S} + 6\hat{\sigma}_{SS}] = 1x10^{-9}$$

Therefore, the Six Sigma-based control limits for standard deviation can be given as:

$$\overline{S} \pm z_{\alpha_S}(K_S)\hat{\sigma}_{SS} \tag{14}$$

Where $z_{\alpha_S}(K_S)$ is the constant such that:

$$P[-z_\alpha(K_S) \le Z \le +z_\alpha(K_S)] = 1-\alpha_{K_R} \tag{15}$$

$$\alpha_{K_S} = (2)(DPMO \text{ corresponding to } K_S)10^{-6}$$

Here, K_S is the current *quality level* with an allowable shift of 1.5 times of standard deviation at which the process is needed to be controlled. For example, if $K_S = 6$, then with shift, we have $DPMO = 3.4$ either on left tail or on right tail. Therefore, $\alpha_{K_S} = 6.8x10^{-6}$ which implies $z_\alpha(K_S) = 4.5$. Therefore, the control limits for Six Sigma-based S-chart become

$$\overline{S} \pm 4.50\,\hat{\sigma}_{SS} \tag{16}$$

and accordingly we have:

$$P[\bar{S} - 4.5\hat{\sigma}_{SS} \leq S \leq \bar{S} + 4.5\hat{\sigma}_{SS}] = 1 - 6.8x10^{-6} = 0.9999932 \quad (17)$$

Therefore, the standard deviation S_i of i^{th} sample is said to be in control if:

$$\bar{S} - 4.5\hat{\sigma}_{SS} \leq S_i \leq \bar{S} + 4.5\hat{\sigma}_{SS} \quad (18)$$

and is out-of-control otherwise.

The computation of the values of either $z_\alpha(K_R)$ or $z_\alpha(K_S)$ with different sigma quality levels is discussed as follows. If the process is operating at three sigma level, then we have the current quality level as $K_R = K_S = 3$. It may be noted that with allowable shift, a three sigma process may result in 66810.63 DPMO. Once this level is maintained, and if there is a scope for improvement, the practitioner may change the value of $z_\alpha(K_R)$ or $z_\alpha(K_S)$. Therefore, various DPMOs, the corresponding $z_\alpha(K_R)$ or $z_\alpha(K_S)$ values are given as shown in Table 1. For more details on computations readers are referred to Harry (1998) and Lucas (2002).

Table 1. Determination of α_{K_R} or α_{K_S} and $z_\alpha(K_R)$ or $z_\alpha(K_S)$

$K = K_R = K_S$	*DPMO*	α_{K_R} *or* α_{K_S}	$z_\alpha(K_R)$ *or* $z_\alpha(K_S)$
3.0	66810.63	0.1336210	1.50
3.5	22750.35	0.0455010	2.00
4.0	6209.70	0.1241900	2.50
4.5	1349.97	0.0027000	3.00
5.0	232.67	0.0004650	3.50
5.5	31.69	0.0000634	4.00
6.0	3.40	0.0000068	4.50

Values given in Table 1 mean that there is flexibility (advantage) in using the proposed Six Sigma-based R- and S-control charts. That is, at any point of time when a process is being monitored, it may not meet the Six Sigma quality level due to higher level of variation. Under this circumstance, the quality practitioner can decide on appropriate $z_\alpha(K_R)$ or $z_\alpha(K_S)$ value in the control charts. It is important to note that the control limits of traditional control charts can be improved to the control limits of the respective Six Sigma-based control charts if $\hat{\sigma}_R = \hat{\sigma}_{RS}$ and $\hat{\sigma}_S = \hat{\sigma}_{SS}$. This is possible if significant reduction in the process variation is achieved. Comparing the capability of the processes from the perspective of specification limits, we notice that:

$$3\hat{\sigma}_R = 4.5\hat{\sigma}_{RS} \quad \Rightarrow \hat{\sigma}_R = 1.5\hat{\sigma}_{RS}$$

and

$$3\hat{\sigma}_S = 4.5\hat{\sigma}_{SS} \quad \Rightarrow \hat{\sigma}_S = 1.5\hat{\sigma}_{SS}$$

This means that while the proposed charts are capable of meeting the specification requirements with minimum possible variability, the traditional charts are not so capable of meeting the same due to higher level of variability.

6. Performance measures for SSQC chart

The performance of any proposed chart is usually studied by means of measures such as ARL, average time-to-signal (ATS), average adjusted time to signal (AATS) and average number of observations to signal

(ANOS) under both in-control and out-of-control situations. For example, readers are referred to Davis et al. (1990), Chakraborti et al. (2008) and Schoonhoven et al. (2009) and Lee (2011). According to Schoonhoven et al. (2009), ARL is often used to describe the likely performance of a control chart as it will indicate quick detection of the out-of-control situation. It may be noted that a large ARL is always desired when the process is stable or in control (Crowder, 1987). The run length of any control chart is the number of samples observed before an out-of-control signal is seen. The occurrence of an out-of-control signal is an indication that some change in the process has occurred due to some assignable cause(s) and hence attention is needed to identify and eliminate those cause(s). Clearly, a control chart is branded as superior, if its ARL is larger than that of the competing chart(s).

In-control situation is referred to the process when there is no shift in the process with respect to the sample statistic being monitored, such as R and $\overline{S}$, and an out-of control situation is referred to the process when there is a shift in the process with respect to $\overline{R}$ and $\overline{S}$. Accordingly, under the assumption of geometric distribution for the number of attempts it takes for an out of signal, ARL can be determined.

6.1. ARL for Six Sigma-based R-chart

Since we used σ_X as the known standard deviation obtained from the specification of the product characteristic X , it may be noted that the Phase I R-chart has been designed in such a way that the fixed false alarm probability (FAP) for Six Sigma-based R-chart denoted by P_{RS} is given as:

$$P_{RS} = P\,[(R_i < \overline{R} - 4.5\hat{\sigma}_{RS})/\sigma_X = d/6] + P[(R_i > \overline{R} + 4.5\hat{\sigma}_{RS})/\sigma_X = d/6] = 6.8x10^{-6} \tag{19}$$

However, in general, the lower limit for range is not considered for out-of-control situation, then the range R_i of i^{th} sample is said to be in control if:

$$R_i > \overline{R} + 4.5\hat{\sigma}_{RS}$$

Therefore, P_{RS} given in (19) becomes:

$$P_{RS} = P[(R_i > \overline{R} + 4.5\hat{\sigma}_{RS})/\sigma_X = d/6] = 3.4x10^{-6} \tag{20}$$

During the monitoring of the process in Phase II, it is known that the state of the process is said to be in-control with Six Sigma quality if each of $\overline{R} - 4.5\hat{\sigma}_{RS} \le R_i \le \overline{R} + 4.5\hat{\sigma}_{RS}$, $i = 1, 2, \cdots\cdots, k$ and the standard deviation $\hat{\sigma}_{II(X)}$ of the process (in Phase II) is equal to the population standard deviation (or assumed standard deviation), that is $\hat{\sigma}_{II(X)} = \sigma_X$ and hence $\hat{\sigma}_{RS} = d_3\hat{\sigma}_{II(X)}$. In our case it is $\sigma_X = d/6$ that is obtained from the specification to ensure Six Sigma quality. Therefore, given $\sigma_X = d/6$, the fixed in-control ARL or the fixed false alarm rate (FAR) of the process can be obtained as:

$$ARL_{ic} = \frac{1}{IP_{RS1}} = \frac{1}{3.4x10^{-6}} = 294117 \tag{21}$$

where IP_{RS1} is the in-control FAP given by:

$$IP_{RS1} = P[(R_i > \overline{R} + 4.5\hat{\sigma}_{RS})/\hat{\sigma}_{II(X)} = \sigma_X] = 3.4x10^{-6} \tag{22a}$$

In case of traditional R-chart, the in-control ARL can be obtained as $1/0.00135 = 740$, since $P[R > \overline{R} + 3\hat{\sigma}_R] = 0.00135$.

As discussed by Lee (2011), when $\hat{\sigma}_{II(X)} \ne \sigma_X$ and hence $\hat{\sigma}_{II(X)} = \gamma\sigma_X$ with $1 < \gamma \le 1.5$, the shifts are defined as small standard deviation shifts and if $\gamma > 1.5$ the shifts are defined as large standard deviation shifts. Here, as suggested by Six Sigma quality program, we allow an allowance for $\hat{\sigma}_{II(X)}$ up to $\pm\gamma\sigma_X$ with $1 < \gamma \le 1.5$ since the process is still in control as long as $\hat{\sigma}_{II(X)} = \pm 1.5\sigma_X$ and hence the ARL under this situation can be obtained as:

$$ARL_{shift} = \frac{1}{IP_{RS2}} \tag{22b}$$

$$IP_{RS2} = P[(R_i > \bar{R} + 4.5\hat{\sigma}_{RS})/\sigma_X < \hat{\sigma}_{II(X)} \le \gamma\sigma_X], \quad 1 < \gamma \le 1.5 \tag{23}$$

Therefore, the out-of-control ARL of the process can be obtained when $\hat{\sigma}_{II(X)} > 1.5\sigma_X$ as follows

$$ARL_{out} = \frac{1}{OP_{RS}} \tag{24}$$

$$OP_{RS} = P[(R_i > \bar{R} + 4.5\hat{\sigma}_{RS})/\hat{\sigma}_{II(X)} > \gamma\sigma_X], \quad \gamma > 1.5 \tag{25}$$

Now using central limit theorem, the values of IP_{RS2} given in (23) for $1 < \gamma \le 1.5$ $(\gamma = 1.25, 1.50)$ and the values of OP_{RS} given in (25) for $\gamma > 1.5$ $(\gamma = 1.75, 2.00, 2.5)$ are obtained. Accordingly, the in-control and out-of-control ARLs given respectively in (22) and (24) are computed.

6.2. ARL for Six Sigma-based S-chart

In case of S-chart, it may be noted that the Phase I S-chart has been designed in such a way that the fixed false alarm probability (FAP) for Six Sigma-based S-chart denoted by P_{SS} is given as:

$$P_{SS} = P[(S_i < \bar{S} - 4.5\hat{\sigma}_{SS})/\sigma_X = d/6] + P[(S_i > \bar{S} + 4.5\hat{\sigma}_{SS})/\sigma_X = d/6] = 6.8x10^{-6} \tag{26}$$

However, in general, the lower limit for standard deviation is not considered for out-of-control situation, then the standard deviation S_i of i^{th} sample is said to be in control if:

$$S_i > \bar{S} + 4.5\hat{\sigma}_{SS}$$

Therefore, P_{SS} given in (26) becomes:

$$P_{SS} = P[(S_i > \bar{S} + 4.5\hat{\sigma}_{SS})/\sigma_X = d/6] = 3.4x10^{-6} \tag{27}$$

During the monitoring of the process in Phase II, it is known that the state of the process is said to be in-control with Six Sigma quality if each of $\bar{S} - 4.5\hat{\sigma}_{SS} \le S_i \le \bar{S} + 4.5\hat{\sigma}_{SS}$, $i = 1, 2, \cdots\cdots, k$ and the standard deviation $\hat{\sigma}_{II(X)}$ of the process (in Phase II) is equal to the population standard deviation (or assumed standard deviation), that is $\hat{\sigma}_{II(X)} = \sigma_X$ and hence $\hat{\sigma}_{SS} = \hat{\sigma}_{II(X)}\sqrt{1 - c_4^2}$. Therefore, given $\hat{\sigma}_{II(X)} = \sigma_X$, the fixed in-control ARL or the FAR of the process can be obtained as:

$$ARL_{ic} = \frac{1}{IP_{SS1}} = \frac{1}{3.4x10^{-6}} = 294117 \tag{28}$$

where IP_{SS1} is the in-control FAP given by:

$$IP_{SS1} = P[(S_i > \bar{S} + 4.5\hat{\sigma}_{SS} / \hat{\sigma}_{II(X)} = \sigma_X] = 3.4x10^{-6} \tag{29}$$

In case of traditional S-chart, the in-control ARL can be obtained as $1/0.00135 = 740$, since $P[S > \bar{S} + 3\hat{\sigma}_S] = 0.00135$.

As suggested by Six Sigma quality program, we allow an allowance up to $\pm\gamma\sigma_X$ with $1<\gamma\leq 1.5$ as the process is still in control as long as $\sigma_X < \hat{\sigma}_{II(X)} \leq 1.5\sigma_X$ and hence the ARL under this situation can be obtained as:

$$ARL_{shift} = \frac{1}{IP_{SS2}} \tag{30}$$

$$IP_{SS2} = P[(S_i > \overline{S} + 4.5\hat{\sigma}_{SS})/\sigma_X < \hat{\sigma}_{II(X)} \leq \gamma\sigma_X], \quad 1<\gamma\leq 1.5 \tag{31}$$

Therefore, the out-of-control ARL of the process can be obtained when $\hat{\sigma}_{II(X)} > \gamma\sigma_X$, $\gamma > 1.5$ as follows:

$$ARL_{out} = \frac{1}{OP_{SS}} \tag{32}$$

$$OP_{SS} = P[(S_i > \overline{S} + 4.5\sigma_{SS})/\hat{\sigma}_{II(X)} > \gamma\sigma_X], \quad \gamma > 1.5 \tag{33}$$

Now using central limit theorem, the values of IP_{SS2} given in (31) are obtained for $1<\gamma\leq 1.5$, $\gamma = (1.25, 1.5)$ and the values of OP_{SS} given in (33) are obtained for $\gamma = (1.75, 2.0, 2.5)$. Accordingly, the in-control and out-of-control ARLs given respectively in (30) and (33) are computed.

Table 2 shows the ARL values $(ARL_{in}, ARL_{shift}$ and $ARL_{out})$ for the proposed Six Sigma-based charts and the traditional charts at different shift levels. Looking at the control limits and the ARL performance, it is clear that the proposed Six Sigma-based R- and S-charts outperform their traditional counterparts. Though the approach to the construction of the proposed Six Sigma control charts is quite different, we applied the same procedure to determine ARL values for the traditional R- and S-charts as well and compared their performances. However, if there are charts with similar approach proposed in future, then effective performance comparison is feasible. The ARL values of traditional charts shown here are close enough to those obtained by Schoonhoven et al. (2011) through simulations when mean range and mean standard deviation are used to estimate unknown population standard deviation. In this article, we have considered numerical examples to demonstrate working of the proposed new charts in comparison with their traditional counterparts. The ARLs are worked out for these examples with an aim to facilitate better understanding of the ARL computation.

Table 2. In-control and out-of-control ARLs with different shift levels

	In control ARL (ARL_{in})	In control ARL with shift (ARL_{shift})		Out-of-control ARL (ARL_{out})		
γ	$\gamma=1$	$\gamma=1.25$	$\gamma=1.5$	$\gamma=1.75$	$\gamma=2.0$	$\gamma=2.5$
Six Sigma Chart	294117	1733	741	336	161	44
Traditional charts	740	25	15	10	7	4

It may be noted that while probability of in-of control signal under the proposed charts is $3.4x10^{-6}$ that results in ARL $=1/(3.4x10^{-6}) = 294117$, in case of traditional charts we have ARL $=1/0.00135 = 740$. Schoonhoven et al (2011) conducted simulations to determine the ARL values of the standard deviation chart when different estimators including average sample range and average standard deviation are used. The different sample sizes (30 and 75) each with subsample of size 5 are considered. The ARLs obtained for all estimators in Schoonhoven et al (2011) are far below than the ARL of the proposed approach when the process is centered. The vast difference is due to the fact that the variability is kept to a minimum in the proposed approach as per the Six Sigma quality requirements. From Table 2, it can be seen that, the ARL value decreases as the shift in the process standard deviation increases.

7. Illustrative Examples

Example 1: R-Chart

Montgomery and Runger (2002) provided data of an extrusion die that is used to produce aluminum rods. The diameter of the rods is considered as is a critical quality characteristic with specification (modified) 35 ± 5 inch. The range values for 20 samples of five rods each are given as 3, 4, 4, 5, 4, 2, 7, 9, 10, 4, 8, 6, 4, 7, 3, 10, 4, 7, 8, 4. Now the control limits can be computed as follows:

Control limits for traditional R-chart:
From (1), it is known that three sigma-based control limits for traditional R-chart are given as

$$\bar{R} \pm 3 d_3 \bar{R} / d_2 =$$
$$5.65 \pm 3 x 0.864 \; x 5.65 / 2.326 \quad \Rightarrow \quad LCL =$$
$$0 \; (-ve), \; UCL = 11.95$$

With central line (CL) = $\bar{R} = 5.65$

Six Sigma-based R-chart
From (5), for a Six Sigma quality, the standard deviation becomes:

$$\hat{\sigma}_{RS} = (d / 6) d_3 =$$
$$(5/6)(0.864) = 0.72$$

Now, using (6) the control limits for Six Sigma-based R-chart are given as

$$\bar{R} \pm 4.50 \; \hat{\sigma}_{RS} =$$
$$5.65 \pm (4.50)(0.72) \quad \Rightarrow LCL = 2.41,$$
$$UCL = 8.89$$

With central line (CL) = $\bar{R} = 5.65$

From Figure 1, it can be seen that according to the traditional R-chart, the process is well within control, whereas, the Six Sigma-based R-chart reveals, the process needs attention with regard to the points 9 and 16 as they are falling outside the UCL and also point 6 that is marginally below the LCL.

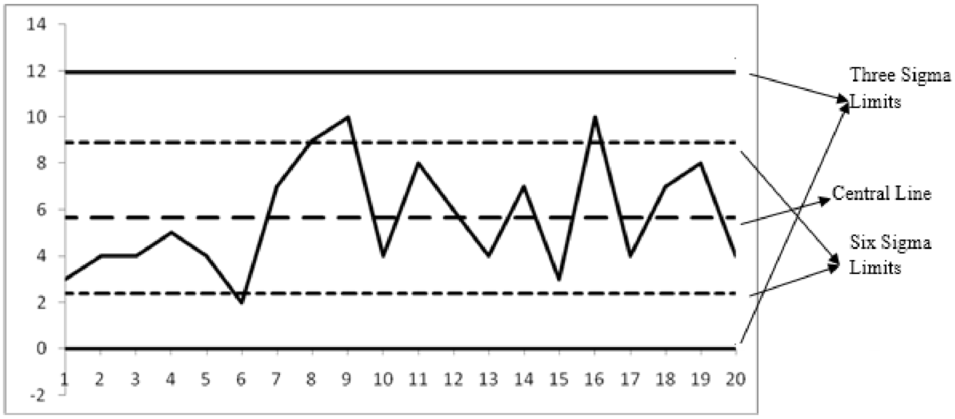

Figure 1. R-chart with traditional and Six Sigma-based control limits (Example 1)

It may be noted that while the process standard deviation obtained from the specification is $\sigma_X = 5/6 = 0.833$, the actual process standard deviation obtained is

$\hat{\sigma}_{II(X)} = \overline{R} / d_2 = 2.43$. This implies that $\hat{\sigma}_{II(X)} = 2.92\,\sigma_X$ with the shift of $\gamma = 2.92$ which is above the allowed shift of 1.5 times of standard deviation. Accordingly, the current sigma level is computed as $K_R = d / \hat{\sigma}_{II(X)} = 5 / 2.43 = 2.05$ which is likely to produce 20182 DPMO. This is an indication that the standard deviation of 2.43 needs to be reduced so that it comes down to 0.833 as required for Six Sigma Quality resulting in 3.4 DPMO.

ARL values for the numerical example of Six Sigma-based R-chart

It is observed that observed the process that the process standard deviation $\hat{\sigma}_{II(X)} = 2.43$ has shifted by $\gamma = 2.92 \quad (> 1.5)$. Therefore, if the out-of-control ARL is computed using Eqns. (24) and (25) as

$$ARL_{out} = \frac{1}{OP_{RS}} = 18$$

$$OP_{RS} =$$
$$P[(R_i > \overline{R} + 4.5\hat{\sigma}_{RS}) / \hat{\sigma}_{II(X)} = 2.92\,\sigma_X] =$$
$$0.05705343$$

Example 2: S-Chart

In the study of controlling the thickness of a film base with specification 180 ± 7 microns, five units each from twenty samples are taken and tested. With an aim to study whether the spread of thickness is under control, it is planned to develop control limits for an S-chart. The standard deviation values are computed and given as 2.35, 4.16, 2.30, 4.87, 5.07, 3.21, 4.39, 3.27, 4.30, 5.03, 5.03, 4.92, 4.51, 5.81, 3.54, 6.23, 6.35, 3.44, 3.13, and 3.21. The control limits are now computed as follows:

Control limits for traditional S-chart:

From (3), it is known that three sigma-based control limits for traditional S-chart are given as:

$$\overline{S} \pm 3\frac{\overline{S}}{c_4}\sqrt{1 - c_4^2} = 4.25 \pm (3)(4.25 / 0.94)\sqrt{1 - 0.94^2}$$

$$LCL = 0(-ve),\ UCL = 8.88$$

With central line (CL) = $\overline{S} = 4.25$

Six Sigma-based S-chart

From (10), for a Six Sigma quality, the standard deviation becomes:

$$\hat{\sigma}_{SS} = (d / 6)\sqrt{1 - c_4^2} =$$
$$(7 / 6)\sqrt{1 - 0.94^2} = 0.1358$$

Now, using (11) the control limits for Six Sigma-based S-chart are given as:

$$\overline{S} \pm z_{\alpha_S}(K_S)\hat{\sigma}_{SS} = 4.25 \pm (4.50)(0.1358)$$
$$\Rightarrow LCL = 3.64,\ UCL = 4.86$$

With central line (CL) = $\overline{S} = 4.25$

From Figure 2, if the traditional S-chart is used, the process looks well within control, whereas, the Six Sigma-based S-chart reveals, the process needs attention with regard to as many points falling outside both the control limits, meaning that the process is not under control in meeting the goal of Six Sigma. It may be noted that while the standard deviation obtained from the specification is $\sigma_X = 7/6 = 1.167$, the actual average process standard deviation is observed as $\hat{\sigma}_{II(X)} = \overline{S} / c_4 = 4.25 / 0.94 = 4.52$. This implies that $\hat{\sigma}_{II(X)} = 3.87\,\sigma_X$ with the shift of $\gamma = 3.87$ which is above the allowed shift of 1.5 times of standard deviation. Accordingly, the current sigma level is computed as $K_S = d / \hat{\sigma}_{II(X)} = 7 / 4.52 = 1.55$ which is likely to produce an alarming 60570 DPMO. Therefore, efforts must be taken to bring down the level of standard deviation close to 1.65 from 4.25 which will reduce the level of DPMO as required for Six Sigma Quality resulting in 3.4 DPMO.

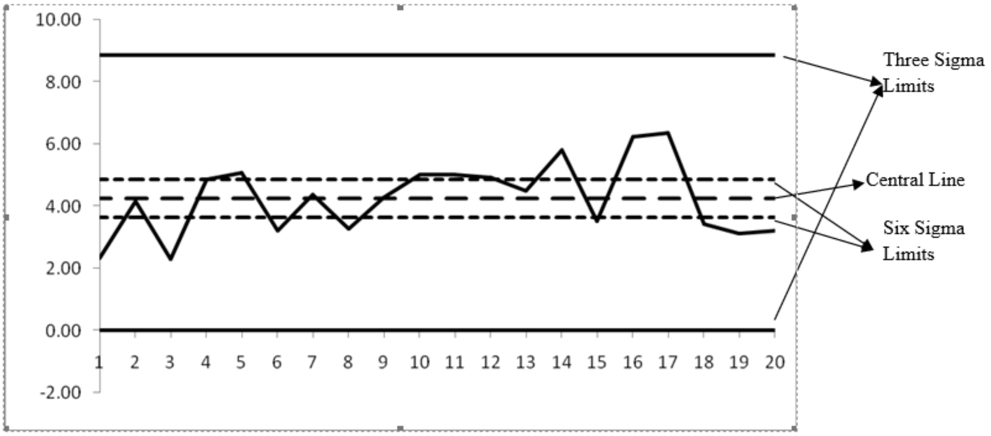

Figure 2. S-chart with traditional and Six Sigma-based control limits (Example 2)

ARL values for the numerical example of Six Sigma-based S-chart

It is observed that observed the process mean range $\bar{S} = 4.25$ has shifted by $\gamma = 2.23 \quad (> 1.5)$. Therefore, if this shifted process standard deviation holds good for the process then the out-of control ARL is computed using Eqns. (31) and (32) as:

$$ARL_{out} = \frac{1}{OP_{SS}} = 4$$

$$OP_{SS} = P[(S_i > \bar{S} + 4.5\hat{\sigma}_{SS}) / \hat{\sigma}_{II(X)} = 3.87\sigma_X] = 0.26434729$$

8. Summary and conclusions

Minimizing variation in a critical process/product quality characteristic is an important objective of any quality improvement program such as TQM and Six Sigma. In fact, too much of variation of such a quality characteristic may result in many undesirable outcomes with respect to scrap/rework costs, customer satisfaction and product output. SPC plays the key role in achieving process improvement though variance reduction for which control charts are found to be effective. These charts are not only useful in detecting unusual signals while monitoring a quality characteristic of a process or product, but they can help in maintaining the quality characteristic well within limits with process mean around the set target. Therefore, it is always preferable to have a control chart that can detect such signals as early as possible. Though Shewhart-type charts are available in the literature, efforts are continuously made to improve the performance of existing charts that can fulfill the present day requirements, mainly due to the domination of industrial automation and information technology. It is observed that, in most of the variable control charts, if the population standard deviation is unknown then the estimation of standard deviation is a challenging task.

As discussed in the literature review, many studies have been made by various authors and practitioners, focusing on obtaining efficient and robust estimators of unknown parameters. In this paper, we proposed a set of new control charts for dispersion- the range and standard deviation charts- from the perspective of Six Sigma quality. The control chart is developed for Phase I situation and the same is maintained to monitor and analyze the performance of the process in Phase II. In these proposed charts, the required standard deviation is obtained from the specification of the measurable characteristic of interest. The multiplication factor for control limits are then obtained by setting the Six Sigma goal of 3.4 DPMO with allowable shift in the mean range of the process. Therefore, is a process is found to be under control when the proposed charts are used, the process can be termed as a Six Sigma process at an appropriate sigma quality level.

The ARL performances of the proposed charts under in-control and out-of control situations are studied in detail. The proposed charts are illustrated using numerical examples. Using the estimated process standard deviation, the current sigma quality levels of the process is also obtained in the respective examples. This helps in knowing the quantum of improvement required not only to bring the process under statistical control, but also to move towards the Six Sigma goal of 3.4 DPMO. From the illustrative examples, it can easily be concluded that the proposed control charts are more efficient than the respective traditional chart in the early detection of out-of-control points. As a future work, it is planned to consider the other variable and attribute control charts from the perspective of Six Sigma quality.

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