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NON-TRADITIONAL MACHINING PROCESS SELECTION – AN INTEGRATED APPROACH

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Abstract: *With a large demand intended for the use of harder and difficult to machine materials like titanium, Inconel, high-strength temperature resistant (HSTR) alloys etc. coupled with the need for high accuracy and desired surface finish have lead us to the situation where we find ourselves entangled in a large pool of Non-Traditional machining (NTM) processes. As such selecting a particular NTM process turns out to be a complicated job for a specific task. Meticulous selection of a NTM process involves a lot of criteria and hence multi-criteria decision making (MCDM) method is used to solve such problems. For the aid of decision maker such that the process of selection gets simplified an integrated method of fuzzy analytic hierarchy process (FAHP) with Quality function deployment (QFD) has been implemented for finding the significance of different technical requirements on a relative basis. Subsequently grey relational analysis (GRA) has been implemented for ranking out the alternatives and it was found that Electrochemical machining (ECM) overrules other NTM processes. A problem already existing in the literature has been picked up for the numerical illustration. The results obtained in the present research study are comparable with the existing literature and sensitivity analysis indicates the robustness of the proposed model.*

Keywords: *NTM process selection, Fuzzy analytic hierarchy process, Quality Function Deployment, Multi-criteria decision making, Grey relational analysis*

1. Introduction

Recent advances in the application of hard and difficult-to-machine materials used in turbine, aviation, tool and die making industries etc. has resulted in the development of Non-Traditional machining processes. Need is felt for machining of specific materials with high precision and advanced surface finish and NTM processes turns out to be extremely useful for such applications as the energy in its direct form is used to remove material from the workpiece. It is worth mentioning that the development of new materials along with innovative and complex product design also tries to test the capabilities of traditional machining methods. Thus the enhanced and efficient process capabilities of NTM processes make them acceptable for the manufacturing industries.

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Since in the last few decades, a large variety of feature rich NTM processes have developed significantly. So for the effective utilization it is very much essential to select a particular NTM process for a desired shape feature and work material combination as it remains a critical issue since the variety of NTM processes is large enough. As such a particular NTM process may be highly acceptable for a given set of requirements but it may fail to prove its acceptability and strength under different set of conditions. Consequently, a cautious approach in the selection of NTM process for a given machining problem is necessary keeping in view the following influential attributes:

1) Physical as well as operational uniqueness of the NTM processes,
2) Potential of machining different shape features on work material,
3) Applicability of diverse processes to different types of materials, and cost factors of various NTM set up

To address the critical issue of careful selection of NTM process, need is felt for developing a method such that best process can be selected for the requirement based application of product. Though there are numerous NTM processes to machine various intricate shape features in different work materials. However, in this paper, seven NTM processes, viz., ultrasonic machining (USM), abrasive jet machining (AJM), electrochemical machining (ECM), electric discharge machining (EDM), electron beam machining (EBM), laser beam machining (LBM) and plasma arc machining (PAM) are considered and they can machine various materials like aluminium, steel, super alloys, titanium ceramics etc. The NTM processes can also make precision holes, standard holes (with slenderness ratio ≤ 20), standard holes (with slenderness ratio ≥ 20), and precision thorough cavities and standard thorough cavities. They can also perform double contouring, surface of revolution, shallow and deep through cutting operations on different work materials.

The presence of a large number of attributes which are not only conflicting in nature but is interdependent also; so the selection of a particular NTM for machining a particular product becomes an intricate decision. Since there are a lot of criteria which are both qualitative and quantitative in nature of which a number of criteria are subjective in nature. So, under such conditions the existence of large number of multi-objective, multi-attribute decision-making methods (MODM/MADM) comes to the rescue of decision maker. There are a number of research paper which have used various MCDM methods like analytic hierarchy process (AHP), analytic network process (ANP), technique for order performance by similarity to ideal solution (TOPSIS), multi-objective optimization using ratio analysis (MOORA) etc. to solve the NTM process selection problem. But there exists a lack of evidence to suggest that a single method combining fuzzy AHP, QFD and grey relational analysis (GRA) have ever been used for solving such a problem. In the existing literature either AHP or QFD alone has been implemented for the NTM process selection (Chakraborty and Dey., 2006; 2007) and calculations are entirely based on the crisp values of the expert’s judgment. But as far as the literature survey is concerned virtually there is no research paper available which has ever applied GRA for solving NTM process selection problem. Also it is worth mentioning that in the existing literature the priority values of the product characteristics have been assigned a definite value which can have a range of priority values. For finding the weight of product characteristics FAHP has been applied since there exists a range of values for it. No doubt there are few papers which do integrate the Fuzzy-AHP and QFD but in the present paper QFD has been implemented for the identification of the technical requirements whereas Fuzzy-AHP has been used to find weightage of individual technical requirements so that problem arising from the traditional QFD model can be avoided. There are a number of criteria which can be either quatitative or qualitative in nature but are essential enough for NTM process selection. Also the inputs to a decision maker at times do come as subjective assessments by the experts (Wang et al., 2000) and in such cases fuzzy theory and GRA becomes essential for dealing with the involved subjectivity. Thus to find out the priority values of the product characteristics from a range of it, to identify the technical requirements combined with need to deal with the subjectivity the present study proposes a methodology which combines fuzzy AHP, QFD and GRA for evaluating and selecting the best possible NTM process.

This paper is organized in the following parts – Section 2 gives a review of the related literatures, while Section 3 is about the Fuzzy AHP (FAHP) method and its involved relations, Section 4 explains the QFD process and its relevance to this paper, Section 5 briefs about the basics of grey theory, Section 6 describes the problem that needs to be solved using the method, Section 7 provides an outline of how the QFD matrix is used to solve the purpose. A methodology has been proposed for solving the problem in Section 8 and Section 9 provides a case study for a numerical illustration and the result and discussion part has been dealt at length in Section 10.

2. Review of the past research

There have been a number of researchers who have used fuzzy methods for solving multi-attribute problems. Since the real life situations involve a lot of uncertainties and grey relational analysis handles it well. The grey theory is based on known degree of information (Julong, 1989) and its advantage over fuzzy theory is that it takes into consideration the condition of the fuzziness (Klir and Yuan, 1995; Zimmerman, 1996). Use of GRA is mostly intended for analyzing a variety of relationships amongst the distinct data sets and also for taking decisions in multi-criterion situations (Hsu et al., 2000, Tong and Wang, 2000).

For NTM process selection past researchers have employed various methods like computer-aided selection procedure and the coding system and used a computer program alongwith a database in backend for the elimination process (Cogun, 1993; 1994). Further, a systematic methodology for NTM selection under conflicting situations incorporating AHP based expert system (Chakraborty and Dey, 2006), combined AHP and technique for order preference by similarity to ideal solution (TOPSIS) method (Yurdakul et al., 2003; Chakladar and Chakraborty, 2008), quality function deployment (QFD)-based expert system (Chakraborty and Dey, 2007) has been investigated. A well planned three level architecture incorporating a front-end a middleware and a database to support the backend for a web-based knowledge base system, digraph-based expert system (Chakladar et al., 2009), ANP (Das and Chakraborty, 2011), MOOSRA method (Chakraborty, 2011) and AHP-TOPSIS based NTM process selection (Choudhury et al., 2013) for selecting the best NTM process has also been investigated. Application, suitability and potential evaluation of mixed data method with the help of three examples for NTM process selection is being reported (Chatterjee and Chakraborty, 2013). A study providing a distinct but systematic approach in fuzzy and crisp environments to deal with a proper selection of the machining process for cutting of carbon structural steel is also investigated by the researchers (Temuçin et al., 2014). Fuzzy MCDM offer greater flexibility for handling complex situations having uncertainty (Soota, 2014). A decision making model with the help of a software to automate the NTM process selection with suitable graphic user interfaces (Prasad and Chakraborty, 2014), Fuzzy AHP and QFD based method for NTM process selection for drilling a hole in aluminum (Roy et al., 2014) and applicability, suitability and computation using operational competitiveness ratings analysis (OCRA) method for solving the nonconventional machining process selection have also been studied (Madic et al., 2015). A large number of literatures have been referred before identifying gaps: Grey relational theory with its strength of solving complicated interrelationship between multiple factors and variables, factor effect evaluation and multiple criteria decision (Chang and Yeh, 2005; Yeh and Lu, 2000) has never been explored for NTM process selection.

The conventional AHP pertains to subjective evaluation of criteria and hence cannot flawlessly imitate the human thinking style. Consequently to evade the involved risks the fuzzy AHP, an extension of AHP, helps in solving the hierarchical fuzzy problems (A. Özdağoğlu and Özdağoğlu, 2007). As such this technique shall be employed in this research study.

Quality function deployment (QFD) offers an organized technique which aims at product planning and development. With the help of it, a product development team can undoubtedly stipulate the requirement of customer in order to evaluate each of the proposed products systematically so as to identify the degree to which it meets the set of customer's requirements.

Therefore, the integration of GRA, FAHP and QFD provides an unique combination of methodologies is being proposed for solving a well-known problem of NTM process selection. The results thus obtained, when compared with the results of the existing literature, validates the usefulness of the approach used in this paper.

3. The Fuzzy AHP (FAHP) Method

Of the various multi-criteria decision-making (MCDM) tools, analytic hierarchy process (AHP) is a structured technique which offers subjective judgement of one criteria over the other and is based on mathematics and psychology. AHP uses eigenvalue approach for the pairwise comparison and was developed by Saaty (Saaty, 1980). The three fundamental steps of AHP are:

1) forming a hierarchy by structuring a decision-making problem,
2) making pairwise comparisons between alternatives and the criteria,
3) synthesizing the priorities for developing an overall evaluation of decision alternatives.

In the recent past a large number of work have been carried out for solving various decision making problems using AHP (Kahraman et al., 2003; Kulak and Kahraman, 2005; Chan et al., 2008, Dağdeviren and Yüksel, 2008). Whereas all these methods stresses on the determination of weights of various influencing attributes and are suitable enough for the analysis of alternatives which have many parameters.

The knowledge and insight of an expert can be dealt in AHP but his thought cannot be perfectly reflected in crisp numbers. In order to overcome the above problem, fuzzy-AHP which integrates the fuzzy theory (Zadeh, 1965) into AHP environment is implemented. To characterize the relative significance among hierarchy's criteria, the fuzzy extension of AHP utilizes a nine level scale of judgments which are expressed through the triangular fuzzy numbers (TFN) (Zhu et al., 1999). Calculations using the TFNs are rather simple and easy. It is helpful also if the available information is subjective and imprecise in a particular decision-making problem (Zimmerman, 1996; Chang and Yeh, 2002; Chang et al., 2007). The simplicity and effectiveness of TFN are useful enough for indicating strength of elements in the hierarchy (Das, 2010). In reality membership function of triangular form is generally used for representing the fuzzy numbers. And it can be represented by a triplet of real numbers K = (a, b, c), where a represents lower bound limit, c the upper bound limit and b being the median value. Existing literature provide a number of scales, but here we are using the one which is rather easy and correspond better to the preference scale of crisp AHP as given in Table 1.

Table 1. Fuzzy scale of preferences (Anagnostopoulos et al., 2007)

Linguistic Variables	Crisp AHP	TFS	Reciprocal TFS
Equally	1	(1,1,1)	(1,1,1)
Equally to Moderately	2	(1,2,3)	(1/3,1/2,1)
Moderately	3	(2,3,4)	(1/4,1/3,1/2)
Moderately to Strongly	4	(3,4,5)	(1/5,1/4,1/3)
Strongly	5	(4,5,6)	(1/6,1/5,1/4)
Strongly to Very Strongly	6	(5,6,7)	(1/7,1/6,1/5)
Very Strongly	7	(6,7,8)	(1/8,1/7,1/6)
Very Strongly to Extremely	8	(7,8,9)	(1/9,1/8,1/7)
Extremely	9	(8,9,9)	(1/9,1/9,1/8)

The fuzzy membership function $\mu_K(y)$ is defined as

$$\mu_K(y) = \begin{cases} 0, & if \ \ y \prec a \\ \dfrac{y-a}{b-a}, & a \le y \le b \\ \dfrac{y-c}{b-c}, & b \le y \le c \\ 0, & if \ \ y \succ c \end{cases} \tag{1}$$

If there are two TFNs K_1 and K_2, where $K_1 = (a_1, b_1, c_1)$ and $K_2 = (a_2, b_2, c_2)$. Then their basic operational laws are as follows:

$$K_1 \oplus K_2 = (a_1, b_1, c_1) \oplus (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2) \tag{2}$$

$$K_1 \otimes K_2 = (a_1, b_1, c_1) \otimes (a_2, b_2, c_2) = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2) \tag{3}$$

$$(a_1, b_1, c_1)^{-1} = (1/c_1, 1/b_1, 1/a_1) \tag{4}$$

Due to the simplicity in usage of extent analysis technique, it has been used for estimating the synthetic degree value in this paper, which was originally introduced by Chang (Chang, 1996). Let $K_{ij} = (a_{ij}, b_{ij}, c_{ij})$ be a triangular fuzzy number (TFN). The steps involved for solving FAHP problem are as follows:

Step1. Attributes are compared pairwise using the fuzzy numbers which are made of lower, mid and upper bound values in a particular level of hierarchy structure as shown in Figure 1.

Step2. The fuzzy synthetic extent value for the 'i 'th object is defined as:

$$SE_i = \sum_{j=1}^{n} K_{ij} \otimes \left[\sum_{i=1}^{m} \sum_{j=1}^{n} K_{ij} \right]^{-1} \tag{5}$$

$$\sum_{j=1}^{n} K_{ij} = \left(\sum_{j=1}^{n} a_{ij}, \sum_{j=1}^{n} b_{ij}, \sum_{j=1}^{n} c_{ij} \right) \tag{6}$$

subjected to:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} K_{ij} = \left(\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} b_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \right) \tag{7}$$

$$\left[\sum_{i=1}^{m}\sum_{j=1}^{n} K_{ij} \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}}, \frac{1}{\sum_{i=1}^{m}\sum_{j=1}^{n} b_{ij}}, \frac{1}{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}} \right) \tag{8}$$

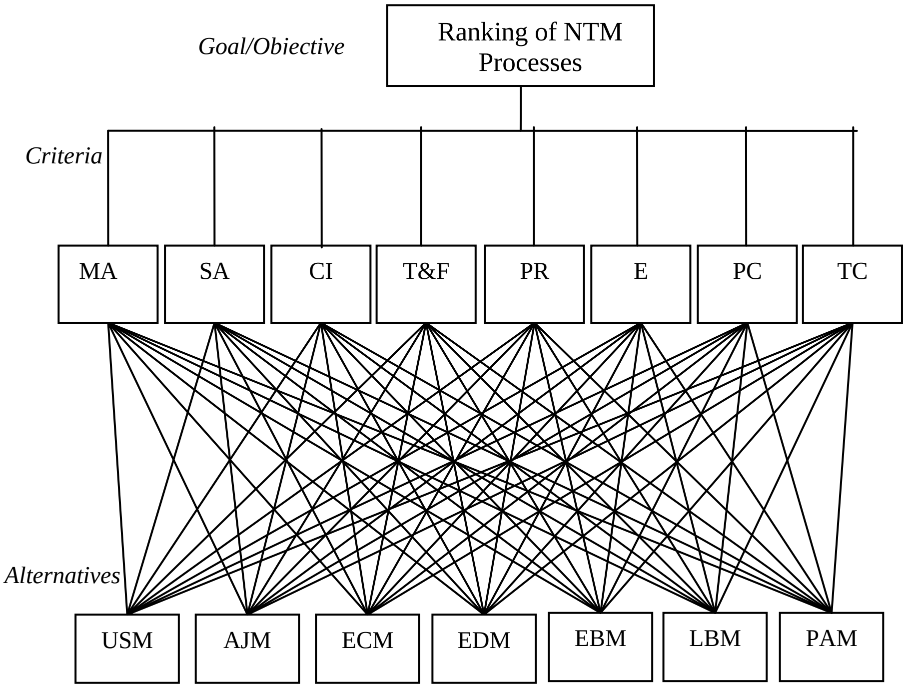

Figure 1. Hierarchy of the criteria

Step3. By comparing the values of SE_i the degree of possibility is then calculated and it can be better expressed as:

$$\begin{aligned} V(SE_j \geq SE_i) &= height\,(SE_i \cap SE_j) = \mu SE_j(z) \\ &= 1\,, if\;\; b_j \geq b_i \\ &= 0\,, if\;\; a_i \geq c_j \\ &= \frac{a_i - c_j}{(b_j - c_j) - (b_i - a_i)}, \; otherwise \end{aligned} \tag{9}$$

The highest intersection point P has p as the ordinate and it lies somewhere in between μSE_j and μSE_i as shown in Figure 2. To compare SE_i and SE_j, both the values of $V(SE_j \geq SE_i)$ and $V(SE_i \geq SE_j)$ are required.

Step4. The minimum degree of possibility $p(i)$ is then calculated as:

$$V(SE_j \geq SE_i) \ \ for\, i, j = 1,2,.....,k$$

$$\begin{aligned} V(SE \geq SE_1, SE_2, SE_3,....., SE_k),\ for\, i = 1,2,3,.....,k \\ = V[(SE \geq SE_1)\, and\, (SE \geq SE_2)\, and .....(SE \geq SE_k)] \\ = \min V(SE \geq SE_i) \ \ for\ \ i = 1,2,3,..........,k \end{aligned} \tag{10}$$

Assuming that:

$$p^*(A_i) = \min V(SE \geq SE_i);\ \ for\ \ i = 1,2,3,......k$$

Then the weight vector is given by:

$$W^* = (p^*(A_1), p^*(A_2),......., p^*(A_n))^T \tag{11}$$

where $A_i\,(i = 1,2,3,.....,n)$ are the n elements.

Step5. With the help of normalization, the normalized weight vectors are then obtained.

$$W = (p(A_1), p(A_2),......., p(A_n))^T \tag{12}$$

where W thus obtained is a non-fuzzy number (Figure 2).

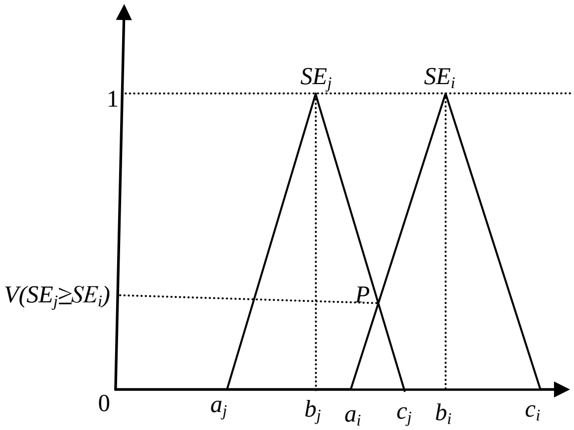

Figure 2. Highest intersection between μSE_j and μSE_i is at point P

4. QFD Process

The concept of Quality function Deployment (QFD) evolved in Japan in late 1960s and early 1970s (Chan and Wu, 2002) and it was practically first implemented by Mitsubishi Industries in 1972, later it was further developed by Akao (Akao, 1990). Other than providing an efficient approach for planning a product and its development based on technological and customer requirement QFD aids in better interpretation of qualitative criteria in objective form which in real effect helps in designing and manufacture of a product.

The well-structured approach of QFD for product planning and development allows the development team to clearly specify the customer's requirement and then to evaluate planned product individually in a systematic manner for a better identification of the degree of satisfaction of customers requirement (Hauser and Clausing, 1988; Wasserman, 1993).

Implementation of QFD starts with the formation of House of Quality (HOQ) matrix as named by Hauser and Clausing (Hauser and Clausing, 1988). The relationship between voice of customers i.e. customer requirement (WHATs) and the quality characteristics or technical requirements (HOWs) (e.g. Chuang, 2001; Chan and Wu, 2002; Govers, 2001) are displayed by HOQ. For the ease of HOQ application the correlation and planning matrices have been removed in the present methodology. A customer-centric and market-focussed approach of decision making emerges when key components of QFD involves WHATs, HOWs and WHYs (Cohen, 1995). Also product characteristics as those corresponding to voices of the customers while process characteristics corresponds to the technical requirement. 'Customer is the king' is a most common proverb that is used in today's competitive world. So nowadays manufacturers put a great thrust to the customer's requirements, and hence a company's job in implementation of QFD is to integrate the customer requirements (CR) to technical requirements (TR) in a feasible manner. To start with the QFD, first list down the TRs which are most likely to affect the CRs. The customer's insight on competitor's product provides help to fix technical targets. And if in case, there remain any discrepancies between the customer's perception and the QFD team's correlation of CR and TR then it can be easily understood by QFD matrix. The response of a company to various customer requirements may be understood by having a look at the vertical part of QFD matrix.

In the proposed paper, customer requirements are considered equivalent to the product characteristics while the technical requirements correspond to the process characteristics.

5. Grey theory

During the last few decades grey theory has gained a wide acceptance in all the fields relevant to human needs. Grey theory was originally developed by Prof. Julong in 1982 and is very effective for solving problems which involves a degree of uncertainty.

The grey system means that the amount of available information is not complete. Since the word 'black' means a total lack of information or no information while 'white' means that the complete information is known. Any grey system can be defined as a system which contains uncertain information presented by grey numbers, grey equations, grey variables, grey matrices etc. If the exact value of variable is unknown but the interval in which it will lie is known then that variable can be termed as a grey number. Say $M=[p,q]$ is an interval number and $M=[p,q]= \{x|\ p\leq x\leq q,\ p\leq q,\ p,q\ \varepsilon\ R\}$. If $M=[p,q]$ and $N=[r,s]$ represent two grey numbers then their basic mathematical operations are given by:

1) $M + N = [p+r,q+s]$
2) $M - N = [p-r,q-s]$
3) $M * N = \min\{pr,ps,qr,qs\},\max\{pr,ps,qr,qs\}]$
4) $M/N=[\min\{p/r,p/s,q/r,q/s\},\max\{p/r,p/s,q/r,q/s\}]$
5) $h*M = [hp,hq]$, where h is a constant.

6. Problem description

In order to validate the concatenated approach, a case study of a large manufacturing company is considered keeping in view that a similar sort of problem for NTM process selection was handled by earlier researchers (Chakraborty and Dey, 2007) where the authors tried to select NTM process based on condition that it can make precision holes, standard holes (with slenderness ratio ≤ 20), standard holes (with slenderness ratio ≥ 20), and precision thorough cavities and standard thorough cavities. They can also perform double contouring, surface of revolution, shallow and deep through cutting operations on different work materials. Also capital investment, tooling and fixtures, power requirement and tool consumption has been considered to be major hurdle for a given material and shape feature combination and hence they have considered as negative influencing factors. Also each NTM needs to satisfy two basic needs of material application and shape application, failing which its selection will be deterred. The two broad factors which forms the basis of analysis using QFD are product characteristics and process characteristics.

6.1. Product characteristics

The various requirements of customer can be properly met if there exists a proper linking of varied characteristics of product which are not only achievable but are independent too. The product characteristics which have been considered in this paper are:

a) Workpiece material (WPM)
b) Shape feature (ShFe)
c) Surface finish (SF)
d) Surface damage depth (SDD)
e) Tolerance (T)
f) Corner Radii (CR)
g) Production Time (PT)
h) Product Economy (PE)

All the above-mentioned product characteristics have been assumed to be independent of each other in order to avoid repetition in analysis.

6.2. Process characteristics

The extent to which the process characteristics of an optimal NTM process meet the desired product characteristic is highly important for its selection. Thus, the process characteristics that are accountable for achieving the required product characteristics are listed as follows:

a) Material Application: It defines the frequency at which a particular NTM process is to be used for a given material.
b) Shape Application: The suitability of specific NTM process for development of a particular shape feature on a specific material.
c) Capital Investment: Involves the net capital expenditure made from installation to operation of NTM process for a requisite application.
d) Tooling and fixtures: For various jobs there could be requirement of change in tooling and the kind of fixture required and these may involve a cost for changing those.
e) Power Requirement: It is power rating of a particular NTM process.
f) Efficiency: It is the ratio of the amount of energy available for material removal on NTM to the amount of input energy supplied to the machine.
g) Process capability: It deals with the capability of a particular NTM process to achieve high precision and surface finish, maximize material removal rate, and minimize surface damage depth. In real sense it can be broken down into five parts viz. surface finish, surface damage depth, tolerance, corner radii and production time that a particular NTM process can provide.
h) Tool consumption: This characteristic takes care of the tool changing requirement, which a few NTM processes may need, for machining a particular product and is also accountable for any cost involved with it.

Thus, after determining all the eight technical requirements and proposed NTM processes pairwise comparison is made among requirements using fuzzy AHP and then QFD is implemented on the weights thus obtained for the technical requirements. QFD has been implemented to take care of the customers (product) requirement. A group decision making tool widely known as Delphi technique has been used for catering the purpose. Further TFNs have been used to tackle the ambiguities, if any, for the purpose of decision making. Calculations using TFNs are somewhat unpretentious keeping in view that TFNs are conducive enough for decision making problems if the available information is subjective and imprecise (Zimmerman, 1996; Chang and Yeh, 2002; Chang et al., 2007). The weights of all the technical requirements have been calculated by integrating fuzzy AHP with QFD.

7. Proposed methodology

The proposed methodology encompasses the integration of FAHP and QFD for determining the weights of all the technical requirements followed with the application of GRA for ranking of the alternatives. The steps for the said purpose are outlined as follows;

Step 1: Identify the various customer requirements i.e. product characteristics.

Step 2: Identify all the technical requirements i.e. process characteristics.

Step 3: By using FAHP the level of importance (normalized weight vectors) of various customer requirements i.e. product characteristics are calculated.

Step 4: Using the expert knowledge of a QFD team a central relationship matrix is constructed

Step 5: Level of importance of all the individual technical requirements i.e. process characteristics are calculated using equation:

$$w_j = \sum_{i=1}^{m} R_{ij} c_i \tag{13}$$

where w_j is level of importance of the jth technical requirement (j=1, 2,….,n); R_{ij} is the quantified relationship between the ith customer requirement and the jth technical criteria in the central relationship matrix; and c_i is the importance weighing of the ith customer requirement.

Step 6: The level of importance of various technical requirement as calculated in previous step is then normalized using the equation:

Step 7: Since the inputs for grey related methods are in intervals. Thus the TFNs can be converted into interval numbers using the α-cut technique (Dubois and Prade, 1980; El-Hawary, 1998) to build membership functions. Also, it better reflects the confidence levels of multiple decision makers (Das 2010). The α-cut technique takes the basis of resolution principle that any fuzzy set A can be retrieved as a union of its α-cuts and also follows the extension principle that:

$[f(A_1\ldots\ldots A_r)]\alpha = f(A_{1\alpha}\ldots\ldots A_{r\alpha})$

By simply changing the value of α different interval can be made. These crisp values are obtained at the intersection points of horizontal α-line with triangular membership functions, for example, points A_1 and A_2 in Figure 3.

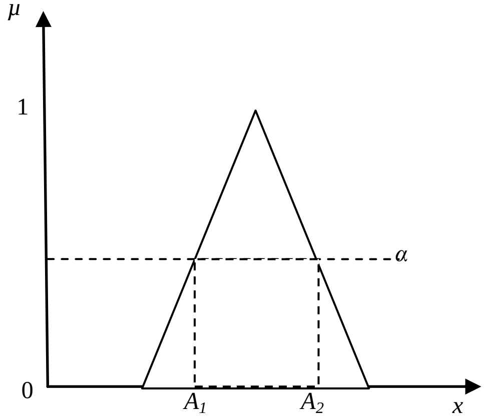

Figure 3. Obtaining Interval data using α-cut technique from TFN

Step 8: The numbers thus obtained in step 7 are then normalized to bring the data in the range of 0-1. The method of normalization also helps in bringing the values to a single dimensionless platform.

Step 9: In this step the weighted interval data is calculated by using the weights as obtained by integration of fuzzy AHP and QFD. Here the criteria weight is multiplied by the performance values of the alternatives under that criterion.

Step 10: The reference number sequences are determined by using the weighted interval number value for each alternative. For a particular criterion and over all alternatives these values are the maximum values of all the lower end values as well as the maximum values of all the higher end values. The values thus obtained represent the maximum weighted value obtained in the data set for that attribute.

In this step grey relational coefficient is to be found out between each alternative and the reference number sequence. For instance, say $X_i(k)$ and $X_j(k)$ are two sequences then their grey relational coefficient ($GRC_{i,j}(k)$) is determined as follows:

$$GRC_{i,j}(k) = \frac{\delta_{\min} + \xi\delta_{\max}}{\delta_j(k) + \xi\delta_{\max}} \qquad (16)$$

where, $\delta_j(k) = \left| X_i(k) - X_j(k) \right|$ represents the absolute difference between two comparing sequences. while, $\delta_{\max} = \max_j \max_l \delta_j(k)$ and $\delta_{\min} = \min_j \min_l \delta_j(k)$, represents the maximum and minimum of the absolute differences of the comparing sequences. The distinguishing coefficient 'ξ' can have value in the range of 0 to 1, and it is the resolving power where lower the value of 'ξ' higher is the resolving power. Each value of $GRC_{i,j}(k)$ thus obtained lies between 0 to 1 and it represents the degree of closeness between two comparing sequences.

Step 11: The grey relational coefficients thus obtained in the previous step are averaged to obtain the grey relational grade ($GRG_{i,j}$), which is computed as follows:

$$GRG_{i,j} = \frac{1}{n}\sum_{k=1}^{n} GRC_{i,j}(k) \qquad (17)$$

where n is the number of responses. The average thus obtained represents the distance of each alternative from the nadir and thus higher value corresponds to the fact that the alternative is close to the optimal.

Step 12: The alternatives are ranked in the descending order of their grey relational grades.

8. Case study for illustration

In order to validate the proposed methodology a case study of a large manufacturing company will now be analyzed. In a company, for improving the productivity simplicity in manufacturing is an essential aspect and in order to achieve that the stress of the company lies in choosing the best suitable NTM process for a combination of feature pertaining to shape that needs to be machined and the work material used. However, in order to find the best alternative of NTM process an expert committee mulls over seven different NTM processes. The processes considered are: Ultrasonic machining (USM), Abrasive jet machining (AJM), Electrochemical machining (ECM), Electric discharge machining (EDM), Electron beam machining (EBM), Laser beam machining (LBM) and Plasma arc machining (PAM). The methodology that has been adopted for the selection of appropriate NTM process has been dealt at length in the following section and is case specific to a company.

Step 1: The product characteristics, i.e. the customer requirements, which have been identified for specific manufacturing processes are workpiece material (WPM), shape feature (ShFe), surface finish (SF), surface damage depth (SDD), tolerance (T), corner radii (CR), production time (PT) and product economy (PE).

Step 2: In the same way, eight major process characteristics, i.e. technical requirements, have been identified which are: material application, shape application, capital investment, tooling and fixtures, power requirement, efficiency, process capability and tool consumption. Further process capability has been further sub-divided in five different aspects viz. surface finish, corner radii, production time, tolerance and surface damage depth.

On the basis of the opinions that have been derived from experts from academia, industry and researchers Table 2, which is a square matrix is formed by inter-comparison of elements in a pairwise fashion. Here a total eight criterias which are product characteristics have been compared to obtain their weightages using the strengths of FAHP. These fuzzy assessment scores have been obtained from experts through the means of questionnaire and converting their ideas in terms of linguistic variables.

Table 2. Fuzzy evaluation of product characteristics (criteria)

Criteria	WPM	ShFe	SF	SDD	T	CR	PT	PE
WPM	(1,1,1)	(1,1,1)	(1,2,3)	(2,3,4)	(1,2,3)	(2,3,4)	(2,3,4)	(6,7,8)
ShFe	(1,1,1)	(1,1,1)	(1,2,3)	(1,2,3)	(1,2,3)	(3,4,5)	(3,4,5)	(4,5,6)
SF	(0.33,0.5,1)	(0.33,0.5,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,2,3)	(1,2,3)	(3,4,5)
SDD	(0.25,0.33,0.5)	(0.33,0.5,1)	(1,1,1)	(1,1,1)	(0.33,0.5,1)	(1,2,3)	(1,2,3)	(0.2,0.25,0.33)
T	(0.33,0.5,1)	(0.33,0.5,1)	(1,1,1)	(1,2,3)	(1,1,1)	(1,2,3)	(1,2,3)	(3,4,5)
CR	(0.25,0.33,0.5)	(0.2,0.25.0.33)	(0.33,0.5,1)	(0.33,0.5,1)	(0.33,0.5,1)	(1,1,1)	(1,1,1)	(1,2,3)
PT	(0.25,0.33,0.5)	(0.2,0.25.0.33)	(0.33,0.5,1)	(0.33,0.5,1)	(0.33,0.5,1)	(1,1,1)	(1,1,1)	(1,2,3)
PE	(0.12,0.14,0.16)	(0.166,0.2,0.25)	(0.2,0.25,0.33)	(3,4,5)	(0.2,0.25,0.33)	(0.33,0.5.1)	(0.33,0.5,1)	(1,1,1)

After this the equations from (5) to (8) are used on the fuzzy evaluation values outlined in Table 2, and hence the TFN values of eight criteria are obtained as follows:

$$SE_1(WPM) = (16,22,28) \otimes (1/126.58, 1/94.59, 1/67.7) = (0.126, 0.232, 0.413)$$
$$SE_2(ShFe) = (15,21,27) \otimes (1/126.58, 1/94.59, 1/67.7) = (0.118, 0.222, 0.398)$$
$$SE_3(SF) = (8.66,12,16) \otimes (1/126.58, 1/94.59, 1/67.7) = (0.068, 0.126, 0.236)$$
$$SE_4(SDD) = (5.11,7.58,10.83) \otimes (1/126.58, 1/94.59, 1/67.7) = (0.04, 0.08, 0.16)$$
$$SE_5(T) = (8.66,13,18) \otimes (1/126.58, 1/94.59, 1/67.7) = (0.068, 0.137, 0.265)$$
$$SE_6(CR) = (4.45,6.08,8.83) \otimes (1/126.58, 1/94.59, 1/67.7) = (0.035, 0.064, 0.13)$$
$$SE_7(PT) = (4.45,6.08,8.83) \otimes (1/126.58, 1/94.59, 1/67.7) = (0.035, 0.064, 0.13)$$
$$SE_8(PE) = (5.35,6.84,9.08) \otimes (1/126.58, 1/94.59, 1/67.7) = (0.042, 0.072, 0.134)$$

Now according to the equation (9) the degree of possibility of $SE_j=(a_j,b_j,c_j) \geq SE_i=(a_i,b_i,c_i)$ is obtained by comparing the values of SE_i as calculated above. All the values of $V(SE_j \geq SE_i)$ are shown in Table 3.

Table 3. Values of $V(SE_j \geq SE_i)$

$V(SE_j \geq SE_i)$	*Value*	$V(SE_j \geq SE_i)$	*Value*	$V(SE_j \geq SE_i)$	*Value*	$V(SE_j \geq SE_i)$	*Value*
$V(SE_1 \geq SE_2)$	1	$V(SE_2 \geq SE_1)$	0.9626	$V(SE_3 \geq SE_1)$	0.5097	$V(SE_4 \geq SE_1)$	0.1806
$V(SE_1 \geq SE_3)$	1	$V(SE_2 \geq SE_3)$	1	$V(SE_3 \geq SE_2)$	0.5532	$V(SE_4 \geq SE_2)$	0.2263
$V(SE_1 \geq SE_4)$	1	$V(SE_2 \geq SE_4)$	1	$V(SE_3 \geq SE_4)$	1	$V(SE_4 \geq SE_3)$	0.6622
$V(SE_1 \geq SE_5)$	1	$V(SE_2 \geq SE_5)$	1	$V(SE_3 \geq SE_5)$	0.9407	$V(SE_4 \geq SE_5)$	0.6151
$V(SE_1 \geq SE_6)$	1	$V(SE_2 \geq SE_6)$	1	$V(SE_3 \geq SE_6)$	1	$V(SE_4 \geq SE_6)$	1
$V(SE_1 \geq SE_7)$	1	$V(SE_2 \geq SE_7)$	1	$V(SE_3 \geq SE_7)$	1	$V(SE_4 \geq SE_7)$	1
$V(SE_1 \geq SE_8)$	1	$V(SE_2 \geq SE_8)$	1	$V(SE_3 \geq SE_8)$	1	$V(SE_4 \geq SE_8)$	1

Table 3. Values of $V(SE_j \geq SE_i)$ (continued)

$V(SE_5 \geq SE_1)$	0.5944	$V(SE_6 \geq SE_1)$	0.02357	$V(SE_7 \geq SE_1)$	0.0235	$V(SE_8 \geq SE_1)$	0.0461
$V(SE_5 \geq SE_2)$	0.6353	$V(SE_6 \geq SE_2)$	0.07051	$V(SE_7 \geq SE_2)$	0.0705	$V(SE_8 \geq SE_2)$	0.0946
$V(SE_5 \geq SE_3)$	1	$V(SE_6 \geq SE_3)$	0.4977	$V(SE_7 \geq SE_3)$	0.4977	$V(SE_8 \geq SE_3)$	0.5464
$V(SE_5 \geq SE_4)$	1	$V(SE_6 \geq SE_4)$	0.8502	$V(SE_7 \geq SE_4)$	0.8502	$V(SE_8 \geq SE_4)$	0.9229
$V(SE_5 \geq SE_6)$	1	$V(SE_6 \geq SE_5)$	0.4588	$V(SE_7 \geq SE_5)$	0.4588	$V(SE_8 \geq SE_5)$	0.5022
$V(SE_5 \geq SE_7)$	1	$V(SE_6 \geq SE_7)$	1	$V(SE_7 \geq SE_6)$	1	$V(SE_8 \geq SE_6)$	1
$V(SE_5 \geq SE_8)$	1	$V(SE_6 \geq SE_8)$	0.9164	$V(SE_7 \geq SE_8)$	0.9164	$V(SE_8 \geq SE_7)$	1

Then the minimum degree of possibility $p^*(i)$ of $V(SE_j \geq SE_i)$ where $i,j=1,2,3,\ldots,k$ is calculated:

$$P^*(1) = \min\ V(SE_1 \geq SE_2, SE_3, SE_4, SE_5, SE_6, SE_7, SE_8) = \min\ (1,1,1,1,1,1,1) = 1$$
$$P^*(2) = \min\ V(SE_2 \geq SE_1, SE_3, SE_4, SE_5, SE_6, SE_7, SE_8) = \min\ (0.9626,1,1,1,1,1,1) = 0.9626$$
$$P^*(3) = \min\ V(SE_3 \geq SE_1, SE_2, SE_4, SE_5, SE_6, SE_7, SE_8) = \min\ (0.5097,0.5532,1,0.9407,1,1,1) = 0.5097$$
$$P^*(4) = \min\ V(SE_4 \geq SE_1, SE_2, SE_3, SE_5, SE_6, SE_7, SE_8) = \min\ (0.1806,0.2263,0.6622,0.6151,1,1,1) = 0.1806$$
$$P^*(5) = \min\ V(SE_5 \geq SE_1, SE_2, SE_3, SE_4, SE_6, SE_7, SE_8) = \min\ (0.5944,0.6353,1,1,1,1,1) = 0.5944$$
$$P^*(6) = \min\ V(SE_6 \geq SE_1, SE_2, SE_3, SE_4, SE_5, SE_7, SE_8) = \min\ (0.0235,0.0705,0.4977,0.8502,0.4588,1,0.9164) = 0.0235$$
$$P^*(7) = \min\ V(SE_7 \geq SE_1, SE_2, SE_3, SE_4, SE_5, SE_6, SE_8) = \min\ (0.0235,0.0705,0.4977,0.8502,0.4588,1,0.9164) = 0.0235$$
$$P^*(8) = \min\ V(SE_8 \geq SE_1, SE_2, SE_3, SE_4, SE_5, SE_6, SE_7) = \min\ (0.0461,0.0946,0.5464,0.9229,0.5022,1,1) = 0.0461$$

The weight vector thus becomes:

$$W^* = (1,0.9626,0.5097,0.1806,0.5944,0.0235,0.0235,0.0461)^T$$

Which after normalizing gives the weights of eight criterias:

$$W = (0.2993,0.2881,0.1525,0.0540,0.1779,0.007,0.007,0.0138)^T$$

Thus, the weights obtained for the eight product characteristics, i.e. the customer requirements viz. WPM, ShFe, SF, SDD, T, CR, PT and PE are 0.2993, 0.2881, 0.1525, 0.0540, 0.1779, 0.007, 0.007 and 0.0138 respectively. The weights thus obtained are non-fuzzy numbers.

Step 3: Using the expert knowledge a central relationship matrix is constructed as detailed in Table 5. The weights of the customer's requirement are used by QFD experts as a scale for finding the weights of technical requirements as detailed in next step.

Step 4: Weights of individual technical requirements are then evaluated with the help of eqn 13 and the outcomes are detailed in Table 5. It is noteworthy to mention that from the production economy point of view capital investment, tooling and fixture, power requirement and tool consumption has been considered to be negatively influencing factors and hence have been assigned the negative values in the QFD matrix. The scale of relationship has been chosen on the basis of relationship that exists between the customer requirements versus technical requirement. Table 4 shows the scale values for the interrelationships existing between customer requirement and technical requirement.

Table 4. Scale of interrelationship

Scale value	Significance
0	No relationship
1	Very weak relationship
3	Slightly weak relationship
5	Moderately Weak
9	Strong relationship

Step 5: The levels of importance of various technical requirements are normalized to obtain degree of importance for the selection criteria i.e. weights for the technical requirements by using the eqn 14.

Step 6: The grey relational method requires the inputs in the form of intervals. While the inputs as obtained from the experts are triangular fuzzy numbers (TFN) as shown in Table 6. The values have been rounded off to two decimal places. Here the criterions are material application(T1), shape application (T2), surface finish (T3), surface damage depth (T4), tolerance (T5), corner radii (T6), production time (T7), capital investment (T8), tooling and fixture (T9), power requirement (T10), efficiency (T11) and tool consumption (T12).

The TFN values of criterions are converted to interval numbers by using α-cut technique, taking α=0.5. For instance, the TFN values for the alternative 3 under criterion 4 is converted to interval value as:

Lower value of the interval = 0.087108 + (0.203517-0.087108)X0.5 = 0.145312
Higher value of the interval = 0.448113 – (0.448113 - 0.203517)X0.5 = 0.325815

And hence the interval value for alternative 2 under criterion 3 is (0.145312,0.325815). In the same manner other interval value has been calculated and is shown in Table 7.

Step 7: Normalization is done to bring the interval numbers in the standard form which is done by dividing the lower end and higher end values of an interval by the highest value under a criterion. For instance the criterion 3 of alternative 3 has been normalized as (0.140782, 0.280195)/(0.167365, 0.339536) = (0.414630, 0.825228). Other values of all the interval falling under different criterions have also been normalized in the same manner and is depicted in Table 8.

Step 8: The weight of the criterions as obtained from step 6 are then used to find the weighted interval data and is obtained by multiplying the criteria weight with the interval data. For instance the weight of criteria 2 as obtained was 0.274546 and the performance interval of alternative 2 under criterion 2 is (0.270729, 0.369613). Thus (0.270729, 0.369613)X0.274546 = (0.074327, 0.101476). In the same manner other interval values have been calculated and is shown in Table 9.

Step 9: The reference number sequence is determined by using Table 9. The reference number sequence is the maximum value of the weighted column in a criterion and is given in Table 10.

Step 10: This step calculates the maximum distance between the reference point and each of the weighted interval value. The maximum distance for each alternative, falling under different criterions, to the ideal is identified as the largest distance calculation. The calculation for alternative 3 under the criterion 4 is Distance_{34}=(0.146803, 0.306875) - (0.116186, 0.260508) = (0.030617, 0.046366). The maximum of these two values is taken i.e 0.046366 in this case. Similarly other distances are calculated and are shown in Table 11.

For a particular alternative the reference point is the minimum of all minima and maximum of all maxima distance. So the reference point for alternative 1 is (0, 0.1902). Table 12 lists the reference points for all the alternatives.

The grey resolving coefficient is then found for all alternatives. The distinguishing coefficient 'ξ' can have value in the range of 0 to 1, so its value is varied from 0.1 to 0.9 with a gradient of 0.1. Table 13 and 14 depicts the value of *GRC* for ξ = 0.1 and 0.2 respectively. Similarly other values of *GRC* for ξ = 0.3 and 0.9. For instance, the GRC_{34} for ξ = 0.1 has been calculated as follows using equation number (16)

$$GRC_{34} = \frac{0 + 0.1 \times 0.2159}{0.0463 + 0.1 \times 0.2159} = 0.3177$$

where (0,0.2159) is the reference point for the alternative.

Step 11: The average of the grey relational coefficients gives total score of the alternatives and is used to rank the alternative. For instance, the extreme right columns of Table 13 and 14 give the average scores for all alternatives for ξ = 0.1 and 0.2 respectively.

Step 12: The alternatives are ranked in the descending order of their average grey relational coefficients as obtained in previous step. Table 15 gives the ranking of alternatives as obtained for different values of 'ξ'.

From Table 15 it can be inferred that the best results are obtained for ξ = 0.2.

Table 5. QFD matrix for determining weights of technical requirements

		Technical Requirement								
		Material Application	Shape Application	Capital Investment	Tooling and Fixture	Power Requirement	Efficiency	Process Capability	Tool Consumption	Scale
Customer Requirement	WorkPiece Material	9	1							0.2993
	Shape Feature	1	9		5					0.2881
	Surface Finish	1						9		0.1525
	Surface Damage Depth							5		0.0540
	Tolerance						1	9		0.1779
	Corner Radii		9		3			9		0.007
	Production Time		5				3	5		0.007
	Production Economy			-9	-5	-3	3		-3	0.0138
Degree of Importance for selection criteria		3.134	2.991	- 0.124	1.393	- 0.041	0.240	3.343	- 0.041	
Normalized degree of importance for selection criteria		28.769	27.454	- 1.141	12.782	-0.38	2.207	30.687	-0.38	

Table 6. The performance of the alternatives according to the Technical Requirements

Criterion	USM	AJM	ECM	EDM
T1	(0.05,0.09,0.16)	(0.03,0.06,0.13)	(0.09,0.15,0.26)	(0.09,0.15,0.26)
T2	(0.08,0.1,0.15)	(0.08,0.1,0.15)	(0.13,0.25,0.43)	(0.08,0.11,0.16)
T3	(0.11,0.23,0.45)	(0.1,0.21,0.41)	(0.09,0.19,0.37)	(0.03,0.06,0.14)
T4	(0.08,0.19,0.42)	(0.12,0.25,0.52)	(0.09,0.20.45)	(0.05,0.13,0.29)
T5	(0.13,0.25,0.45)	(0.05,0.09,0.17)	(0.04,0.07,0.14)	(0.11,0.19,0.38)

Table 6. The performance of the alternatives according to the Technical Requirements (continued)

Criterion	USM	AJM	ECM	EDM
T6	(0.07,0.14,0.27)	(0.11,0.19,0.32)	(0.1,0.17,0.29)	(0.13,0.23,0.38)
T7	(0.1,0.18,0.32)	(0.04,0.07,0.15)	(0.1,0.2,0.37)	(0.1,0.17,0.31)
T8	(0.06,0.11,0.2)	(0.03,0.06,0.11)	(0.1,0.2,0.4)	(0.1,0.19,0.35)
T9	(0.06,0.14,0.32)	(0.05,0.12,0.27)	(0.12,0.24,0.5)	(0.13,0.27,0.54)
T10	(0.09,0.2,0.43)	(0.07,0.16,0.36)	(0.1,0.24,0.5)	(0.06,0.14,0.32)
T11	(0.07,0.15,0.32)	(0.06,0.15,0.28)	(0.11,0.23,0.46)	(0.08,0.15,0.29)
T12	(0.09,0.2,0.45)	(0.06,0.15,0.33)	(0.05,0.12,0.28)	(0.14,0.29,0.59)
Criterion	EBM	LBM	PAM	
T1	(0.09,0.15,0.26)	(0.09,0.15,0.26)	(0.12,0.25,0.47)	
T2	(0.08,0.11,0.16)	(0.08,0.11,0.16)	(0.1,0.22,0.38)	
T3	(0.09,0.17,0.33)	(0.05,0.09,0.2)	(0.03,0.06,0.12)	
T4	(0.04,0.1,0.25)	(0.04,0.08,0.2)	(0.03,0.05,0.12)	
T5	(0.11,0.21,0.37)	(0.06,0.11,0.21)	(0.05,0.08,0.13)	
T6	(0.06,0.1,0.18)	(0.05,0.09,0.15)	(0.06,0.09,0.13)	
T7	(0.04,0.08,0.16)	(0.04,0.07,0.14)	(0.13,0.23,0.38)	
T8	(0.11,0.2,0.37)	(0.1,0.17,0.32)	(0.04,0.07,0.13)	
T9	(0.04,0.1,0.23)	(0.04,0.08,0.18)	(0.03,0.05,0.14)	
T10	(0.08,0.15,0.32)	(0.03,0.07,0.15)	(0.03,0.05,0.11)	
T11	(0.03,0.07,0.14)	(0.03,0.05,0.12)	(0.1,0.22,0.46)	
T12	(0.04,0.1,0.25)	(0.04,0.08,0.2)	(0.03,0.06,0.15)	

Table 7. The performance values of the alternatives in the form of interval data

Alternative	T1	T2	T3	T4	T5	T6
USM	[0.07,0.12]	[0.09,0.13]	[0.17,0.34]	[0.13,0.31]	[0.19,0.35]	[0.11,0.21]
AJM	[0.05,0.09]	[0.09,0.13]	[0.15,0.31]	[0.18,0.38]	[0.07,0.13]	[0.15,0.25]
ECM	[0.12,0.2]	[0.19,0.34]	[0.14,0.28]	[0.15,0.33]	[0.06,0.1]	[0.14,0.23]
EDM	[0.12,0.2]	[0.09,0.13]	[0.05,0.1]	[0.09,0.21]	[0.15,0.29]	[0.18,0.3]
EBM	[0.12,0.2]	[0.09,0.13]	[0.13,0.25]	[0.07,0.18]	[0.16,0.29]	[0.08,0.14]
LBM	[0.12,0.27]	[0.09,0.16]	[0.07,0.23]	[0.06,0.14]	[0.08,0.25]	[0.07,0.16]
PAM	[0.19,0.36]	[0.16,0.3]	[0.04,0.09]	[0.04,0.09]	[0.06,0.11]	[0.07,0.11]
Alternative	T7	T8	T9	T10	T11	T12
USM	[0.14,0.25]	[0.08,0.15]	[0.1,0.23]	[0.14,0.31]	[0.11,0.24]	[0.15,0.33]
AJM	[0.05,0.11]	[0.04,0.09]	[0.09,0.2]	[0.12,0.26]	[0.1,0.21]	[0.1,0.24]
ECM	[0.15,0.28]	[0.15,0.3]	[0.18,0.37]	[0.17,0.37]	[0.17,0.34]	[0.09,0.2]
EDM	[0.14,0.24]	[0.14,0.27]	[0.2,0.4]	[0.1,0.23]	[0.12,0.22]	[0.21,0.44]
EBM	[0.06,0.12]	[0.16,0.29]	[0.07,0.16]	[0.11,0.23]	[0.05,0.1]	[0.07,0.18]
LBM	[0.06,0.11]	[0.13,0.39]	[0.06,0.2]	[0.05,0.18]	[0.04,0.09]	[0.06,0.23]
PAM	[0.18,0.3]	[0.05,0.1]	[0.04,0.09]	[0.04,0.08]	[0.16,0.34]	[0.04,0.1]

Table 8. Matrix showing the standardized performance intervals

Alternative	T1	T2	T3	T4	T5	T6
USM	[0.19,0.34]	[0.27,0.37]	[0.49,1]	[0.35,0.8]	[0.54,1]	[0.35,0.68]
AJM	[0.13,0.26]	[0.27,0.37]	[0.45,0.91]	[0.48,1]	[0.19,0.37]	[0.48,0.82]
ECM	[0.33,0.56]	[0.56,1]	[0.41,0.83]	[0.38,0.85]	[0.16,0.3]	[0.45,0.75]
EDM	[0.33,0.56]	[0.28,0.39]	[0.14,0.3]	[0.23,0.55]	[0.43,0.82]	[0.59,1]
EBM	[0.33,0.56]	[0.28,0.39]	[0.38,0.74]	[0.19,0.46]	[0.45,0.81]	[0.27,0.46]
LBM	[0.33,0.75]	[0.28,0.48]	[0.2,0.66]	[0.15,0.37]	[0.24,0.7]	[0.23,0.53]
PAM	[0.51,1]	[0.46,0.88]	[0.13,0.26]	[0.1,0.22]	[0.18,0.3]	[0.24,0.36]
Alternative	T7	T8	T9	T10	T11	T12
USM	[0.45,0.82]	[0.21,0.4]	[0.25,0.57]	[0.39,0.86]	[0.32,0.69]	[0.33,0.74]
AJM	[0.18,0.36]	[0.11,0.22]	[0.21,0.48]	[0.31,0.71]	[0.28,0.6]	[0.23,0.54]
ECM	[0.5,0.94]	[0.38,0.77]	[0.45,0.92]	[0.46,1]	[0.5,1]	[0.2,0.46]
EDM	[0.45,0.8]	[0.37,0.69]	[0.48,1]	[0.28,0.62]	[0.34,0.64]	[0.48,1]
EBM	[0.19,0.38]	[0.4,0.73]	[0.18,0.4]	[0.3,0.63]	[0.14,0.3]	[0.17,0.4]
LBM	[0.18,0.35]	[0.34,1]	[0.14,0.51]	[0.14,0.48]	[0.11,0.27]	[0.13,0.52]
PAM	[0.6,1]	[0.14,0.26]	[0.1,0.23]	[0.1,0.21]	[0.47,0.99]	[0.1,0.24]

Table 9. Matrix showing the weighted Standardized Interval data

Alternative	T1	T2	T3	T4	T5	T6
USM	[0.05,0.1]	[0.07,0.1]	[0.15,0.31]	[0.11,0.25]	[0.17,0.31]	[0.11,0.21]
AJM	[0.04,0.07]	[0.07,0.1]	[0.14,0.28]	[0.15,0.31]	[0.06,0.11]	[0.15,0.25]
ECM	[0.09,0.16]	[0.15,0.27]	[0.13,0.25]	[0.12,0.26]	[0.05,0.09]	[0.14,0.23]
EDM	[0.09,0.16]	[0.08,0.11]	[0.04,0.09]	[0.07,0.17]	[0.13,0.25]	[0.18,0.31]
EBM	[0.09,0.16]	[0.08,0.11]	[0.12,0.23]	[0.06,0.14]	[0.14,0.25]	[0.08,0.14]
LBM	[0.09,0.22]	[0.08,0.13]	[0.06,0.2]	[0.05,0.11]	[0.07,0.21]	[0.07,0.16]
PAM	[0.15,0.29]	[0.13,0.24]	[0.04,0.08]	[0.03,0.07]	[0.05,0.09]	[0.07,0.11]
Alternative	T7	T8	T9	T10	T11	T12
USM	[0.14,0.25]	[-0.002,-0.004]	[0.03,0.07]	[-0.001,-0.003]	[0.01,0.02]	[-0.001,-0.002]
AJM	[0.05,0.1]	[-0.001,-0.003]	[0.03,0.06]	[-0.001,-0.003]	[0.01,0.01]	[-0.0009,-0.0021]
ECM	[0.15,0.3]	[-0.004,-0.009]	[0.06,0.12]	[-0.002,-0.004]	[0.01,0.02]	[-0.0008,-0.0018]
EDM	[0.14,0.2]	[-0.004,-0.008]	[0.06,0.13]	[-0.001,-0.002]	[0.01,0.01]	[-0.0018,-0.0038]
EBM	[0.06,0.1]	[-0.005,-0.008]	[0.02,0.05]	[-0.001,-0.002]	[0,0.01]	[-0.0006,-0.001]
LBM	[0.06,0.1]	[-0.004,-0.011]	[0.02,0.06]	[0,-0.02]	[0,0.01]	[-0.0005,-0.002]
PAM	[0.18,0.3]	[-0.002,-0.003]	[0.01,0.03]	[0,0]	[0.01,0.02]	[-0.0004,-0.0009]

Table 10. Reference Number sequence

	T1	T2	T3	T4	T5	T6
Max (Min)	0.1475	0.1530	0.1512	0.1468	0.1664	0.1805
Max (Max)	0.2876	0.2745	0.3068	0.3068	0.3068	0.3068
	T7	T8	T9	T10	T11	T12
Max (Min)	0.1836	-0.0012	0.0616	-0.0003	0.011	-0.0003
Max (Max)	0.3068	-0.0025	0.1278	-0.0008	0.022	-0.0008

Table 11. Maximum distances from alternatives to the reference number vector

Alternative	T1	T2	T3	T4	T5	T6
USM	0.190	0.173	0	0.061	0	0.098
AJM	0.213	0.173	0.026	0	0.192	0.053
ECM	0.126	0	0.053	0.046	0.216	0.077
EDM	0.126	0.167	0.215	0.138	0.055	0
EBM	0.126	0.167	0.08	0.166	0.057	0.166
LBM	0.071	0.143	0.103	0.194	0.093	0.144
PAM	0	0.032	0.228	0.238	0.214	0.196
Alternative	T7	T8	T9	T10	T11	T12
USM	0.055	0.001	0.055	0.002	0.006	0.001
AJM	0.195	0	0.065	0.001	0.008	0.001
ECM	0.029	0.006	0.01	0.003	0	0.0008
EDM	0.061	0.005	0	0.001	0.008	0.003
EBM	0.189	0.005	0.076	0.001	0.015	0.0006
LBM	0.198	0.008	0.063	0.001	0.016	0.001
PAM	0	0.0004	0.097	0	0.0006	0

Table 12. Reference points for all the alternatives

Alternative	Min	Max
USM	0	0.1902
AJM	0	0.2135
ECM	0	0.2159
EDM	0	0.2155
EBM	0.0006	0.1891
LBM	0.001	0.1987
PAM	0	0.238

Table 13. Weighted distances to reference point taking $\xi = 0.1$

Alternative	T1	T2	T3	T4	T5	T6	
USM	0.09	0.099	1	0.235	1	0.162	
AJM	0.091	0.11	0.443	1	0.1	0.284	
ECM	0.146	1	0.287	0.318	0.091	0.134	
EDM	0.146	0.114	0.091	0.134	0.282	1	
EBM	0.135	0.105	0.197	0.105	0.257	0.106	
LBM	0.23	0.128	0.17	0.097	0.184	0.127	
PAM	1	0.421	0.094	0.091	0.1	0.108	

Table 13. Weighted distances to reference point taking $\xi = 0.1$ (continued)

Alternative	T7	T8	T9	T10	T11	T12	Avg.
USM	0.256	0.905	0.256	0.886	0.734	0.908	0.544
AJM	0.098	1	0.245	0.918	0.706	0.948	0.495
ECM	0.423	0.776	0.67	0.878	1	0.961	0.564
EDM	0.259	0.801	1	0.933	0.73	0.881	0.531
EBM	0.094	0.79	0.205	0.953	0.567	1	0.376
LBM	0.096	0.726	0.251	1	0.579	0.997	0.382
PAM	1	0.982	0.196	1	0.973	1	0.58

Table 14. Weighted distances to reference point taking $\xi = 0.2$

Alternative	T1	T2	T3	T4	T5	T6	
USM	0.167	0.18	1	0.381	1	0.278	
AJM	0.167	0.198	0.614	1	0.182	0.443	
ECM	0.255	1	0.446	0.482	0.167	0.359	
EDM	0.255	0.205	0.167	0.237	0.439	1	
EBM	0.235	0.187	0.325	0.188	0.405	0.189	
LBM	0.368	0.222	0.285	0.174	0.306	0.222	
PAM	1	0.593	0.173	0.167	0.182	0.195	
Alternative	T7	T8	T9	T10	T11	T12	Avg.
USM	0.408	0.95	0.408	0.94	0.847	0.952	0.626
AJM	0.179	1	0.393	0.957	0.828	0.973	0.578
ECM	0.594	0.874	0.802	0.935	1	0.98	0.658
EDM	0.412	0.889	1	0.965	0.844	0.937	0.613
EBM	0.169	0.881	0.336	0.976	0.721	1	0.468
LBM	0.171	0.838	0.396	1	0.729	0.999	0.476
PAM	1	0.991	0.327	1	0.986	1	0.634

Table 15. Ranking of alternatives as obtained for different values of 'ξ'

Rank	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6	Rank 7
ξ							
0.1	PAM	ECM	USM	EDM	AJM	LBM	EBM
0.2	ECM	PAM	USM	EDM	AJM	LBM	EBM
0.3	ECM	USM	PAM	EDM	AJM	LBM	EBM
0.4	ECM	USM	PAM	EDM	AJM	LBM	EBM
0.5	ECM	USM	EDM	PAM	AJM	LBM	EBM
0.6	ECM	USM	EDM	PAM	AJM	LBM	EBM
0.7	ECM	USM	EDM	PAM	AJM	LBM	EBM
0.8	ECM	USM	EDM	PAM	AJM	LBM	EBM
0.9	ECM	USM	EDM	PAM	AJM	LBM	EBM

9. Result and discussions

The ranks of various NTM processes have been found for different values of 'ξ'. Table 15 gives the ranking of alternatives as obtained for different values of 'ξ'. From this table it can be inferred that ECM is the best suited NTM process for the said application. But when the result thus obtained was compared with result of referred literature (Chakraborty and Dey, 2007) then certain conflicts have been noted.

In the referred literature the ranking of alternatives as performed by authors, using only QFD, are ECM>PAM>USM>EDM>LBM>EBM>AJM, while our ranking for ξ = 0.2 is ECM>PAM>USM>EDM>AJM>LBM>EBM. Thus the ranking AJM, LBM and EBM needs to analyzed in detail since between the literature and our result has two main conflicts and they are:

a) LBM is given as fifth best compared to our ranking which has AJM at fifth place
b) EBM has been said to be sixth best compared to our ranking which places it at the last.

In order to confirm the above-mentioned outcomes, we need to perform certain mathematical analysis and to validate results of present research a sensitivity analysis will be done for the ranking values corresponding to ξ = 0.2.

9.1. Mathematical analysis

The fact that AJM>LBM on the twelve criteria, performance of AJM in seven criteria is better than performance of LBM on same criteria while it is poor on rest five criteria. Now taking the help of priority weights as determined by fuzzy AHP and QFD to perform a short mathematical analysis. In this analysis we multiply the middle value of the performance scores to the priority weights, as:

0.28*(0.06-0.15) + 0.27*(0.1-0.11) + 0.3*(0.21-0.09) + 0.3*(0.25-0.08) + 0.3*(0.09-0.11) + 0.3*(0.19-0.09) + 0.3*(0.07-0.07) + (-0.011)*(0.06-0.17) + 0.12*(0.12-0.08) + (-0.003)*(0.16-0.07) + 0.022*(0.13-0.05) + (-0.003)*(0.15-0.08) = 0.094.

Similarly, for the fact that AJM>EBM on the twelve criteria, performance of AJM in eight criteria is better than performance of EBM on same criteria while it is poor on rest four criteria. Now taking the help of priority weights as determined by fuzzy AHP and QFD to perform a short mathematical analysis. So, on analysis as done above the difference between AJM and EBM comes out to be 0.024.

So the differences between AJM and LBM/EBM comes out to be 0.094 and 0.024, which are not a big difference and hence in our ranking AJM leads both LBM and EBM.

9.2. Sensitivity analysis

A mathematical model as proposed by Bhattacharya (Bhattacharya et al., 2002) will be used for the purpose. The governing equation for it is:

$$SI_i = [(\alpha PFM_i) + (1-\alpha)MRM_i] \qquad (18)$$

where

$$MRM_i = \frac{1}{\left[MRF_i \sum_{i=1}^{n} MRF^{-1}\right]} \qquad (19)$$

where *MRM* is the material removal measure, *MRF* is the material removal factor, *PFM* is the performance factor measure, *SI* is the sensitivity index, α is the performance factor decision weight, and n is the number of alternatives.

The PFM values, i.e. the priority value of each alternative is taken as the performance measure value as obtained from average grey relational grade corresponding to ξ = 0.2. The *MRF* values are the standard material removal rate (MRR) values for the different alternatives, and *MRM*s have been designed in such a way that, as shown in equation (19), to obtain a non-dimensional quantity. By doing this we can combine MRR values, i.e. cardinal measures, with the *PFM* values, i.e. ordinal measure, as shown in equation (18). The units of *MRF* are mm^3/min, whereas *MRM* values are non-dimensional entities. Selection of proper value of α is an important issue which needs to be jointly decided by design engineer, production manager and maintenance engineer. The value of α determines the degree to which importance is provided among the measures of material removal and performance factor by the judgement maker and thereby its effect can be properly analyzed on the sensitivity plot. So by using equation (18) variation of SI with α can be plotted for all alternatives as shown in Figure 4.

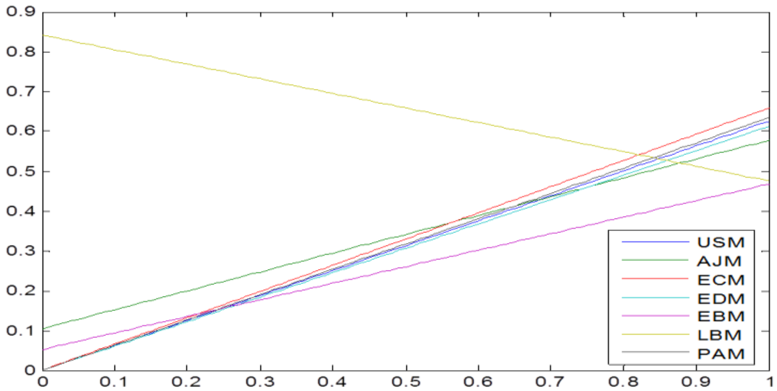

Figure 4. Graphical display of alternatives based on sensitivity analysis

In Figure 4, the x-axis represents the variation in value of α while the y-axis represents the variation in value of *SI*. So from this plot and for α > 0.8 the order of NTM processes are similar to that found by us using fuzzy AHP, QFD and GRA i.e. ECM>PAM>USM>EDM>AJM>LBM>EB M.

10. Conclusions

In the present analysis, the researchers have used the specialties of fuzzy AHP, amalgamated with the benefits that can be derived from QFD for determining the weightages of various technical and customer requirements. However a final ranking of the various NTM processes have been evaluated using the grey theory. The fuzzification process provides a greater flexibility to the decision makers by taking care of uncertainties. However the proposed hybrid and integrated method has never been reported before for the said problem and it's an essence of present research. The future research scope may include the development of an expert system that incorporates the present methodology, however the methodology can be further developed and fuzzified parameters can be used in QFD too for adding a greater flexibility. Last but not the least, the present methodology can also be tried in some other form of selection processes viz. material selection, robot selection etc., and it can be a future scope of present work.

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