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SIX SIGMA-BASED X-BAR CONTROL CHART FOR CONTINUOUS QUALITY IMPROVEMENT

Abstract: *Introduced by Shewhart, the traditional variable control chart for mean (X-bar Chart) is an effective tool for controlling and monitoring processes. Notwithstanding, the main disadvantage of X-bar chart is that the population standard deviation is unknown though the sample mean is an unbiased estimator of the population mean. There are many approaches to estimating the unknown standard deviation with the expertise available with the researchers and practitioners that may lead to varying conclusions. In this paper, an innovative approach is introduced to estimate the population standard deviation from the perspective of Six Sigma quality for the construction of the proposed control chart for mean called Six Sigma-based X-bar control chart. Under the assumption that the process is normal, in the proposed chart the population mean and standard deviation are drawn from the process specification from the perspective of Six Sigma quality. After discussing the aspects of the traditional X-bar control chart, the procedure for the construction of the proposed new Six Sigma-based X-bar control chart is presented. The new chart is capable of maintaining the process mean close to the target by variance reduction resulting in quality improvement. Also, it may be noted that at a point of time, the process, though under statistical control, may be maintaining a particular sigma quality level only while the goal is Six Sigma quality level of just 3.4 defects per million opportunities. Hence, as a practice of continuous quality improvement, it is suggested to use the proposed control chart every time with improvement till the goal of Six Sigma with 3.4 defects per million opportunities is achieved. An innovative cyclic approach for performing the continuous quality improvement activity is also presented. The construction of the proposed Six Sigma-based X-bar control chart is demonstrated using an illustrative example.*

Keywords: *continuous quality improvement, conventional control chart, statistical process control, target range, Six Sigma-based X-bar control chart, upper and lower quality limits*

1. Introduction

Today's competitive scenario forces organizations to adapt and implement one or more of quality improvement programs such as quality circles, total quality management (TQM), Six Sigma, ISO quality system standards etc. There are various quality awards such as Malcolm Baldrige National Quality Award (MBNQA) and European Quality Award (EQA) that recognize organizations that have successfully implemented quality improvement initiative and achieved business excellence. The famous quality gurus like Crosby (1979), Deming (1982) and Juran (1988) preached the aspects of quality management and quality philosophy that will lead to quality improvement and business excellence as well.

Clearly, the various philosophies, programs, methodologies, and awards intend to promote a common goal of developing an integrated total quality program by engaging in continuous improvement. In fact, continuous quality improvement plays a critical role in achieving and maintaining high standards of quality of processes and hence the products. Essentially, continuous improvement of quality of a process or product is nothing but the continuous reduction of variation about a target. This calls for a process or product characteristic to be around the target with minimum variation which may result in achieving high quality processes and products, cost savings and other preset goals (Andersson et al., 2006). In statistical process control (SPC), control charts play a major role in achieving the goal of sticking to the target with minimum variation leading to quality improvement.

Ever since the introduction of control charts by Shewhart (1931), quality practitioners have been showing enormous interest in making use of these charts for controlling and monitoring of processes and products. While standard quality control charts have wider applications and are used by quality control practitioners over many decades, in the recent past there has been an enhanced interest in proposing more sophisticated methods for SPC (Box and Narasimhan, 2010). Most of these sophisticated charts, such as cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts, aim for the detection of shifts in process mean from its target as early as possible. For example, some recent studies made on such advanced control charts related to different areas of interest can be found in (Gadre and Rattihalli, 2004; Ferrer, 2007; Hsu *et al.*, 2009; Hassan *et al.*, 2010; Ryan and Woodall, 2010; Ryu *et al.*, 2010; Reynolds and Lou, 2010; Zhu and Lin, 2010). Similarly, a number of studies have been reported on the use of control charts for quality improvement in specific areas such as manufacturing, healthcare and other services (e.g. Finison and Finison, 1996; Hawkins and Olwell, 1998).

Given a set of subsamples with observed values from a normal process with mean μ and standard deviation σ, in a traditional Shewhart-type control chart for process mean (called the X-bar chart), the upper and lower control limits are placed at $\pm 3\hat{\sigma}$, where $\hat{\sigma}$ is an estimate of the unknown population standard deviation σ. Clearly, the main disadvantage of this approach is the unavailability of the population standard deviation. Many authors have suggested different estimators for σ and studied their performances. Under the assumption of normality, Chakraborti *et al.* (2008) studied the use of three different estimators of the population standard deviation while the overall sample mean is used as an estimator of the population mean. Schoonhoven et al. (2009) considered five unbiased estimators of population standard deviation and studied the aspects of out-of-signal probabilities. Radhakrishnan and Balamurugan in many of

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their work (For e.g., Radhakrishnan and Balamurugan, 2010; Radhakrishnan and Balamurugan, 2011) have proposed new control limits (for charts based on exponentially weighted moving average and number of defects etc.) from the perspective Six Sigma Quality (SSQ) initiative taking into account the standard deviation is determined in terms of known process tolerance and process capability index. However, in SPC applications, it is advisable to determine the capability of the process only after ensuring that the process is under control.

In his work related to SSQ, Ravichandran (2006) highlighted that while control charts are useful in monitoring process stability, there exist many parameters in a process that need to be monitored from the perspective of their respective specifications. Further, Ravichandran, (2006) proposed the concept of *setting up quality specification* from the perspective of SSQ where "target range" covering the ± 1.5 times of shift in the standard deviation is introduced. Unlike the case in the traditional control chart that is used to control and monitor the process and product, for a quality improvement activity, it is opined that the reduction of process variance is of prime concern given an allowable shift of $\pm 1.5\sigma$ (McFadden, 1993) in the process mean. This directly prompts for the reduction of process variation that can result in the SSQ level of just 3.4 defects per million opportunities (DPMO) by quality improvement teams. However, one cannot achieve 3.4 DPMO in one stretch but it is possible by means of continuous quality improvement.

Keeping this aspect in mind, a new control chart for mean (X-bar control chart) called Six Sigma-based X-bar control chart is proposed from the perspective of SSQ that helps in achieving reduced process variation with mean centered at the target through continuous quality improvement. The proposed Six Sigma-based X-bar control chart is innovatively designed in such a way that can control and monitor the process continuously in a cyclic manner. It can detect any shift in the process mean faster than the traditional three sigma-based X-bar chart. It also checks if the shift in the process mean is within the allowable target range of plus or minus 1.5 sigma. We notice that to the best of our knowledge there have been no major studies done linking the SSQ in developing the proposed Six Sigma-based X-bar control chart for process mean.

The remainder of this paper is organized as follows. In Section 2, the concept of Six Sigma and its role in continuous quality improvement program is presented. In Section 3, the procedure for construction of the proposed Six Sigma-based X-bar chart is described. An example is considered to illustrate the working of the proposed control chart in Section 4. The case of traditional three-sigma control chart is also given in this section. The paper concludes with a summary and some discussions in Section 5.

2. Six Sigma, Control Chart and Quality Improvement

Six Sigma is a customer driven approach that represents the systematic implementation of various statistical methods, tools and techniques for quality improvement and hence for customer satisfaction. Obviously, Six Sigma is all about variance reduction, cost reduction and higher performance. A typical SSQ program can be implemented using define measure, analyze, improve and control (DMAIC) approach. An organization is said to be successful in the implementation of Six Sigma program, if it can achieve zero defects processes which is more or less equivalent to achieving 3.4 DPMO. According to (Harry, 1998; Lucas, 2002; Ravichandran, 2006a), an organization can be classified as either world class, industrial average or noncompetitive based on the sigma quality level (SQL) it has achieved at a point of time. Here, *world class* organizations maintain SQL between 5 sigma to 6 sigma, *industrial average* organizations maintain SQL between 4 sigma to 5 sigma and *noncompetitive* organizations are able to maintain SQL below 3 sigma.

Since no organization can achieve the SSQ level of 3.4 DPMO in one attempt and it can be at a particular SQL at a point of time (Harry, 1998; Lucas, 2002). Therefore, it is important to continuously improve the process by means of variance reduction and moving process mean closer and closer so that next SQL can be achieved and this process can be continued until the goal of 3.4 DPMO is met. One of the important statistical methods used in the implementation of Six Sigma program is SPC. As discussed earlier, control charts play a major role in controlling and monitoring of a process by means of reduced variance and moving the process average closer to the target. A control chart is said to be more efficient, if it can detect any shift in the process average (i.e., change in the process) faster than any other chart. Therefore, if the present SQL is known, then we suggest to using the proposed Six Sigma-based control chart to achieve further improvement in quality and then this cycle continues until the goal is met. This Six Sigma – control chart-quality improvement cycle is depicted in Figure 1.

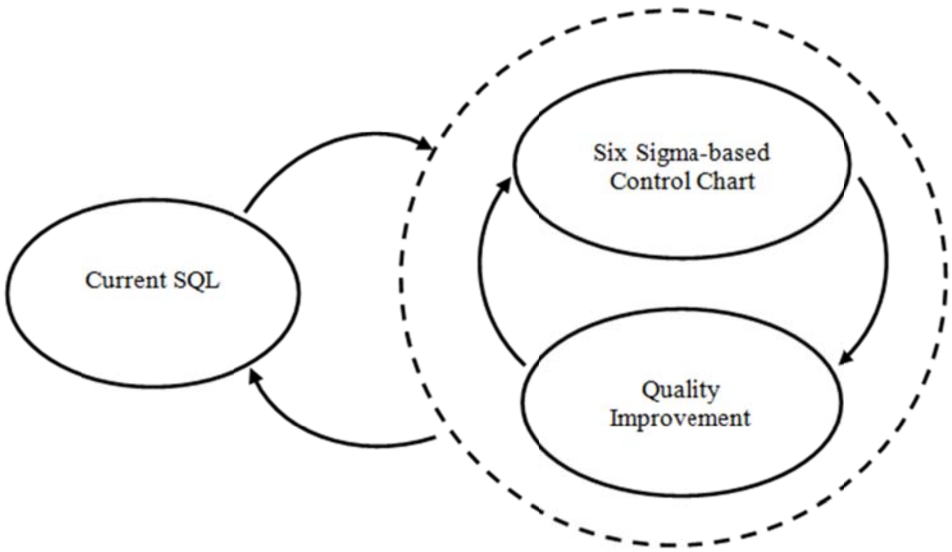

Figure 1. Six Sigma – control chart-quality improvement cycle

For example, if the current SQL is 3.5 sigma then the organization may be interested in improving this SQL. This can be achieved by the proposed Six Sigma-based Control chart by knowing the level of shift in the process mean from the target and the process variation. Through quality improvement activity efforts can be made to move the process mean closer to the target by finding the means for variance reduction. The application of the proposed chart and the quality improvement activity are treated successful, if the next SQL is anything more than 3.5 sigma (refer to the illustrative example section).

3. Six Sigma-based X-bar Control Chart

It may be noted that the concept of Six Sigma that relies on normal distribution allows a shift, though not desirable, in the process mean up to ± 1.5 times of standard deviation from the target (process mean) still results in just 3.4 DPMO (McFadden, 1993; Lucas, 2002). Ravichandran (2006) introduced the concept of *setting up quality specification* from the perspective of SSQ in which the *target range* covering the ± 1.5 times of shift in the standard deviation is considered. Using these ideas, in this paper it is attempted to develop a new and innovative control chart called Six Sigma-based X-bar control chart for process mean that can outperform the traditional control chart in case of variables. In this regard, the process or product specification itself is considered as a normal population of the variable of interest. The population standard deviation is obtained following the procedure suggested by Ravichandran (2006).

Following Shewhart (1931), it is well known that when the process parameters μ (mean) and σ (standard deviation) are unknown then they are estimated as $\hat{\mu}$ and $\hat{\sigma}$ from the historical data. Since this historical data is limited, according to Schoonhoven *et al.* (2009), the traditional control limits $\hat{\mu} \pm 3\hat{\sigma}/\sqrt{n}$ used for sample means do not perform like the control limits $\mu \pm 3\sigma/\sqrt{n}$. Schoonhoven *et al.* (2009) further argued that a solution to this problem is to correct the control limits by replacing the fixed constant 3 by $c(n,k,p)$, and set the control limits as $\hat{\mu} \pm c(n,k,p)\hat{\sigma}/\sqrt{n}$ where $c(n,k\ p)$ denotes the factor that is dependent on the number of samples k, the sample size n and the probability of out of control signal p. As discussed earlier, many authors such as Chakraborti *et al.* (2008) and Schoonhoven *et al.* (2009) have proposed control limits based on the use of different types of estimators of σ as well.

In this article, given k samples, each of size n, with means $\bar{x}_1, \bar{x}_2, \cdots\cdots, \bar{x}_k$ we propose the control limits

$$\hat{\mu} \pm z_\alpha(K_c)\hat{\sigma}_{ss}/\sqrt{n} \qquad (1)$$

for controlling the sample means, where:

$\hat{\mu} = \frac{1}{k}\sum_{i=1}^{k}\bar{x}_i$, $\bar{x}_i = \frac{1}{n}\sum_{j=1}^{n}x_{ij}$, x_{ij} is the j^{th} observation i^{th} of sample,

$$P[-z_\alpha(K_c) \le Z \le +z_\alpha(K_c)] = 1-\alpha_{K_c} \qquad (2)$$

$$\alpha_{K_c} = (2)(\text{DPMO corresponding to } K_c)10^{-6} \qquad (3)$$

and Z is the standard normal variate. Here, K_c represents the current SQL at which the process is needed to be controlled. For example, if $K_c = 6$, then we have $DPMO = 3.4$ either on left tail or on right tail. Therefore, $\alpha_{K_c} = (6.8)10^{-6}$ which implies $z_\alpha(K_c) = 4.50$. Further, in (1), $\hat{\sigma}_{ss}/\sqrt{n}$ is the estimated standard deviation associated with $\hat{\mu}$ from the perspective of SSQ. We propose to obtain $\hat{\sigma}_{ss}$ from the perspective of SSQ as follows:

Let us consider a measurable characteristic, say X, that follows normal process with mean $T=\mu$ and variance σ^2. Since not all values of X towards tails are acceptable, the specification of X is usually given in the form $T \pm K\sigma$, where T is the *target* or population mean, K is a positive constant and σ is the population standard deviation. Notationally, we have $X \sim N(T,\sigma^2)$ and $P(T-K\sigma \le X \le T+K\sigma) = 1-\alpha_K$ where α_K is a prespecified probability value such that $\alpha_K = P(X<T-K\sigma)+P(X>T+K\sigma)$ From $T \pm K\sigma$, we get half of the process spread as $K\sigma = d$ (say) which implies $\sigma = d/K$ and hence we have $\hat{\sigma}_{ss} = \sigma = d/K$.

Therefore, we have $\frac{\hat{\sigma}_{ss}}{\sqrt{n}} = \frac{d/K}{\sqrt{n}}$.

It is evident that since d is fixed, as sigma (σ) decreases the constant (K) increases and vice-versa. Accordingly, the constant K

is the SQL of the process with respect to the process quality characteristic X. The limits $T - K\sigma$ and $T + K\sigma$ are respectively well known as the *lower specification limit* (LSL) and *upper specification limit* (USL). Hence, for a typical SSQ process we have $K = 6$ and hence $\hat{\sigma}_{ss} = d/6$. Therefore, the required Six Sigma-based control limits become

$$\hat{\mu} \pm (4.50)\left(\frac{d/6}{\sqrt{n}}\right) = \hat{\mu} \pm (4.50)\left(\frac{\hat{\sigma}_{ss}}{\sqrt{n}}\right) \qquad (4)$$

It may be noted that, according to McFadden (1993) there can be displacement (shift) in the average by ± 1.5 times of standard deviation over long periods of time. Though undesirable, such a shift up to ± 1.5 times of standard deviation still results in just 3.4 DPMO. Therefore, as long as the estimated mean $\hat{\mu}$ is within $T \pm 1.5\sigma$ then *process shift* is said to be under control. Now, $T \pm 1.5\sigma$ is called *target range* (Ravichandran, 2006). According to Six Sigma we have $K = 6$, and hence $P(X \geq T + 6\sigma) = 3.4x10^{-6}$ if the shift is on the right side (i.e., $T \leq \hat{\mu} \leq T + 1.5\sigma$) and $P(X \leq T - 6\sigma) = 3.4x10^{-6}$ if the shift is on the left side (i.e., $T - 1.5\sigma \leq \hat{\mu} \leq T$). It may be noted that if $\hat{\mu} = T$, then the process is said to be a centered process and in this case we have $P(X \geq T + 6\sigma) = 1x10^{-9}$ and $P(X \leq T - 6\sigma) = 0.1x10^{-9}$.

Now, the computation of the values of $z_{\alpha}(K_c)$ with different SQLs is discussed as follows. If the process is operating at three sigma level, then we have the current quality level as $K_c = 3$. It may be noted that with allowable shift, a three sigma process may result in 66810.63 DPMO. Once this level is maintained, and if there is a scope for improvement (Refer to Figure 1), the practitioner may change the value of $z_{\alpha}(K_c)$. Therefore, various DPMOs, the corresponding $z_{\alpha}(K_c)$ values are given as shown in Table 1 (Harry, 1998; Lucas 2002).

Table 1. Determination of α_{K_c} and $z_{\alpha}(K_c)$

$K = K_c$	*DPMO*	α_{K_c}	$z_{\alpha}(K_c)$
3.0	66810.63	0.1336210	1.50
3.5	22750.35	0.0455010	2.00
4.0	6209.70	0.1241900	2.50
4.5	1349.97	0.0027000	3.00
5.0	232.67	0.0004650	3.50
5.5	31.69	0.0000634	4.00
6.0	3.40	0.0000068	4.50

However, a typical traditional three sigma control chart for means have the control limits as

$$\hat{\mu} \pm 3\left(\frac{\hat{\sigma}}{\sqrt{n}}\right) = \overline{\overline{X}} \pm 3\left(\frac{\overline{R}/d_2}{\sqrt{n}}\right) = \overline{\overline{X}} \pm A_2\overline{R} \qquad (5)$$

where $\overline{R} = \frac{1}{k}\sum_{i=1}^{k} R_i$ is the average range of the subsample ranges R_i , $i = 1, 2, \cdots\cdots, k$. The constant A_2 is tabulated in most text books for the given subsample size n. For more details refer to Ravichandran (2010).

4. Illustrative Example

Consider the situation where it is of interest to control the process with regard to the thickness of a transparent film for which the specification is given as 180 ± 7. For a Six Sigma quality, we have $\hat{\sigma}_{ss} = d/6 = 7/6 = 1.166667$. Assuming that the given population is normal with mean 180 and standard deviation 1.166667 , twenty subsamples are drawn each of size five. The means and ranges for these 20 subsamples are given below: means:184.0, 179.6, 184.4, 179.8, 179.2, 181.4, 178.4, 183.8, 180.0, 178.6, 179.6, 182.8, 182.4, 180.8, 178.0, 182.6, 178.6, 181.4, 181.4, 178.4; ranges: 6, 10, 5,11, 13, 8, 11, 8, 12, 12, 14, 12, 9, 15, 8, 15, 15, 9, 8, 8. From this data, the grand mean $\hat{\mu}$ or $\overline{\overline{X}}$ is computed as 180.76 and the average range $\overline{R}$ as 10.45. Accordingly, the Six Sigma-based control limits and the traditional three sigma control limits are obtained as>

Six Sigma-based control limits

$$\hat{\mu} \pm (4.50)\left(\frac{\hat{\sigma}_{ss}}{\sqrt{n}}\right) = 180.76 \pm (4.50)(1.66667)/\sqrt{5} \Rightarrow LCL = 178.41, \quad UCL = 183.10$$

With central line $\overline{\overline{X}} = 180.76$

Figure 2 depicts both the proposed Six Sigma-based and traditional three sigma-based control limits (lower control limit, LCL; upper control limit UCL; central line). The twenty subsample means are plotted on it. It can be seen from Figure 2 that few samples (sample numbers 1,3,8 and 15) fall outside the Six Sigma-based control limits (out-of-control signals) whereas the same are shown as 'in -control' points under the traditional three sigma control limits. Clearly the proposed Six Sigma-based control chart is more efficient one than the traditional three sigma control chart.

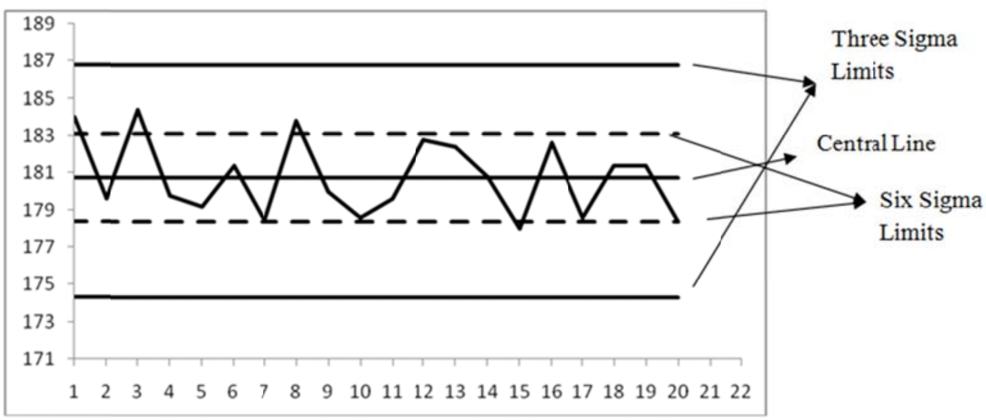

Figure 2. Six Sigma-based and traditional X-bar control charts

Aspects to be considered for Quality Improvement

Referring to Figure 1, the activities under SSQ program do not stop here with the mere development of Six Sigma-based control chart and it calls for further analysis to look out for the avenues for quality improvement. In this example, it may be noted that while the target is $T = 180.76$, the actual process mean is observed as $\hat{\mu} = 180.76$. This shows a shift of (180.76-180.00)/1.166667=0.6514 times of standard deviation on the right side which is well within the allowed shift of 1.5 times of standard deviation. At the same time, the process standard deviation computed from the data is $S = 2.04$, this implies that the SQL of the '*centered process*' is $K_c = d/S = 7/2.04 = 3.43$ times of standard deviations with 302 DPMO. However, incorporating the actual shift of 0.6514, the current SQL becomes $3.43 - 0.6514 = 2.78$ which will result in 2718 DPMO. This means that there is a possibility of 1359 DPMO on either side.

This gives $\alpha_{K_c} = 2(1359)10^{-6} = 0.002718$ which implies $z_\alpha(K_c) = 2.78$ meaning that one has to go a long way to reach $z_\alpha(K_c) = 4.50$ which is equivalent to the SQL of 6 (Refer to Table 1).

Table 2. Process Situation and Required Improvement

	Average	Standard deviation	$K = K_c$	$DPMO$	α_{K_c}	$z_\alpha(K_c)$
Population	$\mu = T = 180$	$\sigma_{ss} = 1.166667$	6.00	3.4	0.0000068	4.50
Process	$\overline{\overline{X}} = 180.76$	$S = 2.04$	3.43	1359.0	0.0027180	2.78
Improvement Required (Reduce by)	0.76	0.87333	2.57	1355.6	0.00271732	1.72
	Activities		Results			

This is an indication that the standard deviation of 2.04 needs to be reduced so that it comes down to 1.166667 as required for SSQ resulting in 3.4 DPMO. This reduction will also substantiate the fact that the processes mean 180.76 can be moved towards the target 180.00. The details are consolidated in Table 2.

5. Summary and Conclusions

It is well known that the traditional variable control chart for mean (X-bar Chart) is an effective tool for controlling and monitoring processes. However, in using this traditional X-bar chart, the main disadvantage is that the population standard deviation is unknown though the sample mean is an unbiased estimator of the population mean. There exist many conventional ways of estimating population standard deviation such as sample standard deviation, sample range etc. In this paper attention has been paid to the aspect of estimating the unknown standard deviation. We have considered an innovative approach to estimate the population standard deviation from the perspective of SSQ for the construction of the proposed Six Sigma-based X-bar control chart for mean. Since the population standard deviation is not known, the specification itself it treated as the population from which the population standard deviation is obtained.

Further the multiplication factor $z_\alpha(K_c)$ is obtained from the SSQ perspective and hence the proposed control chart is found to be more efficient than the traditional three sigma control chart. The procedure also suggest for a cyclic approach to achieve quality improvement after checking the SQL at the end of every cycle. That is, unlike the traditional control chart, the proposed new chart helps to know the status of the process in terms of SQL and DPMO. The new chart is capable of maintaining the process mean close to the target by variance reduction resulting in quality improvement. This in turn helps to involve in continuous quality improvement activities so that the process can achieve the Six Sigma quality's goal of 3.4 DPMO. Hence, as a practice of continuous quality improvement, it is suggested to use the proposed control chart every time with improvement till the goal of Six Sigma with 3.4 DPMO is achieved.

The construction of the proposed Six Sigma-based X-bar control chart is demonstrated using an illustrative example. The aspects to be considered for quality improvement are also provided in this example. It can be understood from the example that how the proposed procedure works as an innovative approach to quality improvement in a cyclic way.

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